# Asymptopia

CENTRE FOR MATHEMATICAL SCIENCES NEWSLETTER

UNIVERSITY OF CAMBRIDGE

October 2007


## After graduation

By Geoffrey Grimmett, Head of DPMMS and Peter Haynes, Head of DAMTP

Around 500 students have graduated from Cambridge Mathematics since the last issue of Asymptopia and we extend to them our very best wishes for the future. Each year, around 200 undergraduates complete Part II of the Mathematical Tripos and receive their BAs, and similar numbers complete Part III as a self-contained graduate-level course. There are nearly 50 students each year on our two MPhil courses, in Statistical Science and Computational Biology, and nearly 60 completed PhDs in the last twelve months.

The numbers, and the international mix of our graduate courses, are impressive, but that is only part of the story. An astonishing amount of energy is invested in the selection for admission of the very best individuals, those with the strongest potential to benefit from what Cambridge Mathematics offers. "Admission" is not a precise science, but we follow the principle that the best mathematics is blind to superficial descriptors such as social and educational backgrounds.

Our first Open Day for alumni and friends "since CMS" took place on 22 September, and a wonderful occasion it was. Frank Kelly spoke on the mathematics of road pricing, Imre Leader on (frighteningly) large numbers, and Neil Turok on the Big Bang. There were presentations by Julia Hawkins on the Millennium Mathematics Project and Ray Goldstein and on mathematical biology. Finally, Stephen Hawking held us in his thrall with his personal story.

The Centre for Mathematical Sciences has itself attracted further recent attention through the award to our architect, Ted Cullinan, of the 2008 RIBA Royal Gold Medal. The Medal is approved personally by the Queen and is given annually to a person or group of people whose influence on architecture has had a truly international effect. We offer our sincere congratulations to Ted.

The forthcoming Research Assessment Exercise (RAE) has caused a frenzy of activity across the UK University sector. All research activity since 2001 will be assessed, and the outcomes will have a major influence on research funding for some years to come. Cambridge Mathematics has done very well in previous RAEs, but the rules shift, and this is a ball to watch very carefully. It is now accepted that the RAE has evolved into an unsustainable monster, and there are strong calls from Whitehall for change towards a formulaic approach based on research grant income. The UK mathematics community has argued successfully that such an approach, in an unmoderated form, would be bad for our subject.

We finish on personal matters. First, hearty congratulations to our ex-Schlumberger Professor Mahadevan, now at Harvard, on the award of an Ig Nobel Prize (honouring scientific achievements that "first make people laugh, and then make them think") for his work (done in Cambridge, UK) on the wrinkling of sheets. And finally, after five years, one of us, Geoffrey Grimmett, is handing over the baton as Head of DPMMS to Martin Hyland. Martin works in logic, category theory, and theoretical computer science.

GARETH MARLOW

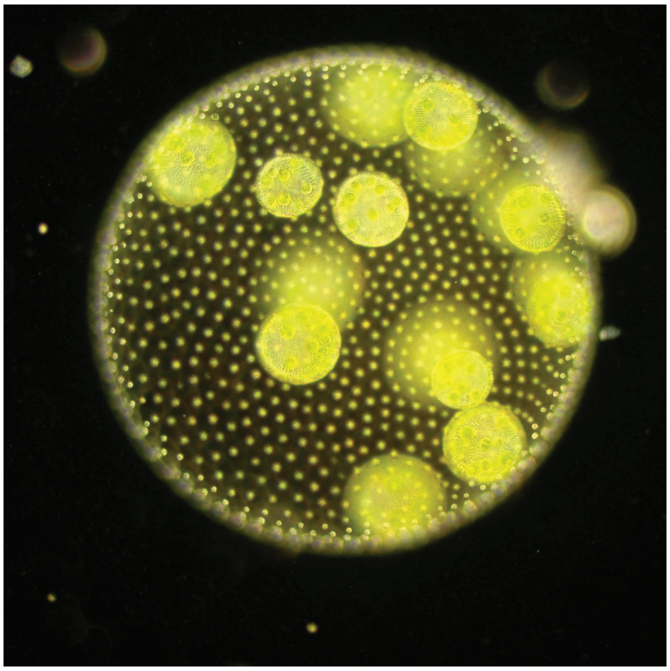
Alumni at the CMS.

## Physical aspects of the evolution of biological complexity

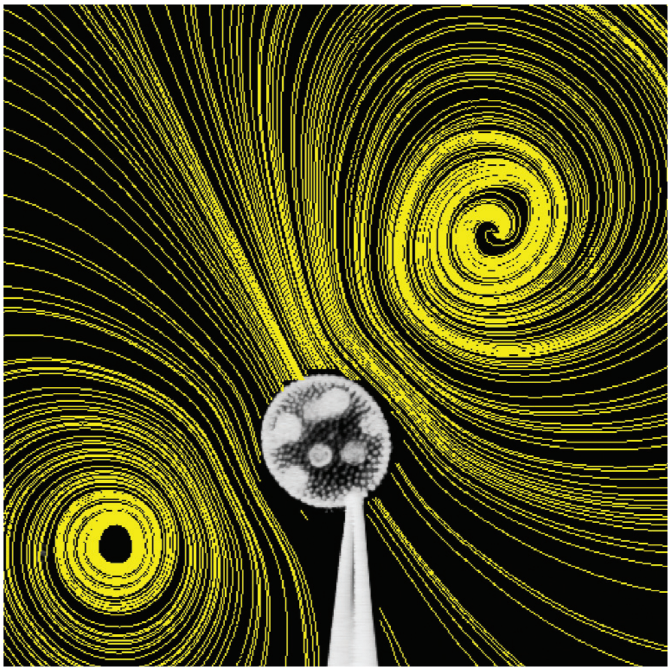
By Ray Goldstein

Some of the most fascinating questions in biology concern the evolutionary transitions which led from unicellular organisms to multicellular ones, and the development of complex, differentiated behaviour among cells. A new research group in DAMTP is investigating these and related questions in the laboratory and through mathematical and physical modelling.

Late in Antony van Leeuwenhoek's life, long after he made his great contributions to the microscope and started a revolution in biology, he peered into a drop of pond water and discovered



The freshwater alga *Volvox*.



A glass micropipette holding an algal colony.

one of nature's geometrical marvels, the freshwater alga which, years later, Linneaus named *Volvox*. This photosynthetic colonial alga has a spherical body plan, with thousands of individual cells on its surface, each with two hair-like appendages called flagella, and spherical daughter colonies growing inside. This species is closely related to a single-cell organism called *Chlamydomonas*, which is like one of the flagellated surface cells of *Volvox*. In fact, there is a whole set of species in between, the smaller ones having only surface cells, which are capable of both reproduction and swimming, the larger ones exhibiting specialisation into those two functions. At some point in this series, nature has decided to exhibit cell differentiation, a division of labour not unlike what economists tell us makes for greater productivity. This particular system is the "hydrogen atom" of the multicellular differentiation problem, displaying one of the most elementary forms of specialisation in a geometry that gives new meaning to the "spherical cow" metaphor.

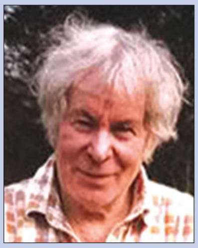
Biologists have long asked about the origins of multicellularity, often phrasing the central question as: What are the advantages of increasing size and specialisation? For microorganisms living in water, exchanging nutrients and metabolites with their environment, it is perhaps not surprising that the physics of buoyancy, diffusion, and mixing are central to this question. Many of the historically important research directions within DAMTP, even on the much larger scales of the ocean and the atmosphere, relate to similar issues.

The example outlined above is one of a set of very basic questions in evolutionary and organismal biology that is the focus of our new Biological Physics research group. We subscribe to the philosophy for which DAMTP is so well known, that the best theoretical work often comes from close ties to experiment. So we have spent the past year building up new laboratories on the lower ground floor of Pavilion H which house a broad range of facilities to study these problems experimentally, and thus to provide the all-important grist for our theoretical mills. Our labs include high-tech growth chambers and incubators for algae and bacteria, a clean room in which we use photolithography methods to produce microfluidic chambers for the study of biological flows, and a set of three microscopy suites in which we use fluorescence imaging, mechanical micromanipulation, and optical trapping to investigate these systems. Some of the methods we use to characterise fluid flows are very similar to those used on much larger scales elsewhere in the G.K. Batchelor Laboratory, such as particle imaging velocimetry (PIV). These techniques can reveal quite striking features of the flagella-driven flows around algal colonies. An example is shown here, in which a glass micropipette holds an algal colony while microspheres are advected by the surface flagella. The measured streamlines indicate the very large scale over which the flow extends. Indeed, these organisms live in the world of very large Péclet numbers (where advection dominates diffusion), a regime whose mathematical treatment is quite rich and nontrivial. Of particular interest are the scaling laws which relate exchange of passive scalars to the Péclet number, as these impact directly on the fundamental question of the advantages of size.

The development of complexity within larger organisms also involves the generation of important fluid flows *within* cells, typically seen as the phenomenon of "cytoplasmic streaming" – the persistent circulation of the cell's contents driven by the action of motor proteins which move in great numbers, entraining fluid. Discovered over two centuries ago by the Italian physicist Bonaventura Corti, cytoplasmic streaming has long been conjectured to play an important role in metabolism, especially in homeostasis. While there is circumstantial evidence that this is the case, surprisingly little quantitative information has been obtained on the interplay between this fluid dynamical phenomenon and cellular metabolism, particularly with regard to mixing. Unravelling the mystery of why nature has engineered these flows and how they relate to metabolism and growth will be another major focus of our group.

Ray Goldstein joined DAMTP in September 2006 as the Schlumberger Professor of Complex Physical Systems.

## People at the CMS
## Peter Whittle



Peter Whittle is Emeritus Professor of Mathematics for Operational Research.

My interest in stochastic processes began during my undergraduate studies in New Zealand, when summer vacation work in the NZ Department of Scientific and Industrial Research introduced me to a range of fascinating applications. When in 1949 the time came for me to embark on my PhD studies I was accepted at Cambridge, Manchester, Stockholm and Uppsala. I opted for the last, attracted by the possibility of an immediate entry into research, plus also the glow of Nordic romanticism and running prowess.

My doctoral work concerned time series analysis, essentially constructing a complete theory for the linear Gaussian case. This led to a Docent appointment at Uppsala, but after two years I returned to my post held open for me at NZDSIR.

Then ensued a uniquely enriching period. Eager colleagues introduced me to the nonlinear models of ecology, the optimisation of oceanographic detector arrays, the random media of geophysics, polymer statistics – glancing but significant encounters. I felt that I had mined the obvious lode in time series analysis, but veins ran from it to spatial processes, prediction and stochastic control – these last two topics being thoroughly penetrated by optimality considerations.

A visiting year (1957) at the Australian National University led me to apply for a vacant lectureship in the Cambridge Statistical Laboratory (henceforth, the Lab), which had been hesitantly absorbed by the Faculty of Mathematics, and where I found a very happy and stimulating group. In 1961 I was honoured to become Maurice Bartlett's successor in the Manchester chair of mathematical statistics. Here my work in polymers and stochastic control took a promising turn, and I found new optimisation problems in the sequential inference procedures we all employ all the time in an informal way.

My increasing interest in optimisation made it natural for me to apply for the Churchill Chair in Mathematics for Operational Research, newly founded in Cambridge by a generous donation from Esso. On my arrival in 1967 I found that I was expected to deliver the new wave in applicable mathematics. In fact I constructed and delivered three new Part II courses in the Mathematical Tripos, on optimisation and communication theory. This stretched me almost irreversibly – I should have done Part III, after all! However, the courses have survived, mutated and developed as an essential part of applied mathematics in our time. My probability text bucked convention by axiomatising expectation, but also proved a survivor.

My interest in stochastic control continued, culminating in a two-volume text and the solution of some problems that had acquired classic status. One of these was the celebrated multi-armed bandit problem, essentially concerned with the optimal dynamic distribution of effort under uncertainty. John Gittins (at the time in the University of Cambridge Engineering department) had in fact developed the key insight in the mid 1960s. However, recognition of this advance was slow, until an improved proof I derived in 1979 proved catalytic.

Another example is that of optimal risk-sensitive control, which I solved for the hitherto resistant case of imperfect state observation. One is essentially choosing a performance measure which implies a degree of optimism or pessimism on the part of the optimiser. It emerges that there is effectively a "ghost in the machine", who turns the problem into a two-person game by either aiding or opposing the optimiser according as one supposes optimism or pessimism. The pessimistic extreme turned out to be identifiable with the fashionable topic of H-infinity control.

My early interest in queuing networks was taken up powerfully by Frank Kelly, who concentrated on communication applications. The innovatory quality of his research group is recognised in both British Telecom and the USA. The network theme has proved a King Charles's head in my own case: a continuing preoccupation with random graphs and polymer models, a substantial diversion into artificial neural nets, and a recent text on the structural optimisation and evolution of networks.

This account rather overlooks the discontinuity of my retirement in 1994, a discontinuity smoothed by a kindly host: the Lab, in the first instance.

# Understanding random growth

By James Norris

The growth of a cluster over time can create beautiful and apparently chaotic patterns. Think for example of the spread of a lichen over the surface of a rock. Although mathematics cannot hope to predict the shape of any individual cluster, there are ways in which it may be able to contribute to our understanding of the behaviour of a typical cluster. It may also help to account for different sorts of pattern in terms of different mechanisms of growth.

The progressive nature of growth invites the creation of computer simulations, based on a simple basic step, perhaps randomised to reflect an unpredictable natural environment. Much work of this type has been done, with considerable success in reproducing naturally occurring phenomena. The simulation in Figure 1 illustrates a particularly simple model, in which stick-like objects, wandering randomly in the plane, stick to the cluster, one after the other. The colours reveal the time at which particles arrived, the red particles being first. Notice how the cluster grows by the development of fingers, which have a fractal-type structure: this is a common feature in growth models.

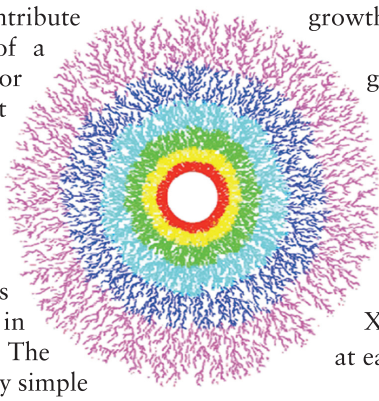


Figure 1: A simulation of cluster growth

A further step in understanding cluster growth would be to characterise the large-scale behaviour of the cluster in terms of elementary random processes which we already understand in detail. Examples of such processes would be the simple symmetric random walk, or its scaling limit, Brownian motion. The random walk is a sequence of random numbers $X_1, X_2, \ldots$, which increases or decreases by 1 at each step with probability 1/2. When the steps are taken very frequently and the size of the steps is suitably reduced to compensate, the random walk converges to a continuous random path, which is called Brownian motion.

Such an analysis of cluster growth has generally proved hard to achieve but the dynamics of the illustrated cluster are sufficiently simple to make this possible. In work with Amanda Turner, a former PhD student at the CMS and now a lecturer at the University of Lancaster, we studied this model using complex analysis and in particular the theory of conformal maps. We could identify the cluster with an essentially unique conformal map defined on a fixed set S, obtained by removing from the plane the unit disc centred at the origin. The map takes the boundary of the disc to the much more complicated boundary of the cluster, and has the property that, for any segment of the disc boundary, of length $L/(2\pi)$ say, the probability that a randomly diffusing particle will hit the corresponding part of the cluster is exactly $L$. The cluster is then encoded in a composition of randomly rotated basic conformal maps defined on S. We can show that the size of each of the fingers of the cluster, measured in terms of the corresponding segment of the disc boundary, evolves as a Brownian motion, stopped when it hits 0. Work continues on a more comprehensive description of the cluster in terms of an infinite family of coalescing Brownian motions.

This is just one example of how a conjunction of stochastic analysis with complex analysis is opening up our understanding of scaling limits for random planar structures. The (much deeper) theory of stochastic Loewner evolutions, developed by Lawler, Schramm, Werner and others, is another example, which applies to many different structures in two-dimensional statistical physics. In the past fifty years the most powerful techniques in analysing randomness were associated with a one-dimensional parameter, usually interpreted as time. Now it seems we may also get to grips with two dimensions!

James Norris is Professor of Stochastic Analysis at DPMMS.

# European Women in Mathematics 2007

EWM delegates at the CMS.

The 13th general meeting of European Women in Mathematics, EWM 2007, took place during 3-6 September at the CMS. An impressive number of 85 women from 25 countries attended the meeting. Talks at EWM 2007 covered a wide range of mathematical areas with enthusiastic presentations by, amongst others, Dusa McDuff from Stonybrook, USA, on symplectic geometry, Ramdorai Sujatha from Mumbai (and winner of the 2006 Ramanujan Prize) on number theory, and Vera Sos from Budapest on combinatorics, as well as the CMS's Toni Beardon speaking on mathematics education and Natalia Berloff on quantum fluids.

In addition to scientific talks, there were discussions on the future role of EWM, a discourse on mathematics in developing countries, as well as splinter and poster sessions given by young mathematicians.

"You don't often see a gathering of women professors like there is [at the EWM 2007]," said Caroline Series, Professor of Mathematics at the University of Warwick and founding member of EWM. "We have some of the very top women mathematicians in the world here, and it's really inspiring to meet them personally."

# News at the CMS

Over the summer a number of awards and prizes have been bestowed on CMS members. The London Mathematical Society has awarded the Whitehead Prize for mathematicians under the age of 40 to Ivan Smith for his work on symplectic topology and to Oliver Riordan for his work on graph theory. The Senior Whitehead Prize went to Béla Bollobás for his defining – and sometimes redefining – work on combinatorics. Further honours from the LMS went to Michael Green who has been awarded the Naylor Prize and Lectureship in Applied Mathematics in recognition of his founding work in superstring theory.

Artur Ekert, recently departed from the CMS for Oxford and Singapore, has been awarded the Hughes Medal for his work on quantum cryptography and quantum computation. Robert Gramacy has received the Savage Award for an outstanding doctoral dissertation and Teruyoshi Yoshida has been awarded a Clay Mathematics Institute Research Fellowship. Ben Green has received the Sastra Ramanujan Prize for his work on number theory.

We would also like to congratulate Nick Dorey and John Rallison on their promotions to Professor, and Ben Allanach and Peter Friz on their promotion to Reader.

A warm welcome goes to our new arrivals Peter Markowich, Professor of Applied Mathematics, Phil Dawid, Professor of Statistics, David Spiegelhalter, the first Winton Professor of the Public Understanding of Risk, and Harvey Reall, Lecturer in Theoretical Physics.

To discuss any aspect of making a donation in support of mathematics at Cambridge, please contact Patrick Hawke-Smith (ph250@cam.ac.uk)