Other news from the CMS

Professor John D Barrow FRS of DAMTP has won the 2006 Templeton Prize for progress towards research or discoveries about spiritual realities. The Templeton Prize was founded in 1972 by Sir John Templeton and is awarded annually to encourage the advancement of knowledge in spiritual matters. It honours Professor Barrow's work on the relationship between life and the universe and the nature of human understanding.

Professor Steve Brooks of DPMMS has won the 2005 Guy Medal in Bronze. The Guy Medals, named after distinguished statistician William Guy FRS, are awarded by the Royal Statistical Society, who honoured Dr Brooks for his "deep contributions to the assessment of convergence of Markov chain Monte Carlo methods and for his application of modern Bayesian methods to important topics in many fields but especially in population ecology".

The 2005 Adams Prize has been awarded to Professor Jonathan Sherratt of the Department of Mathematics, Heriot-Watt University, Edinburgh. The prize, named after the mathematician John Couch Adams, is awarded jointly each year by the Faculty of Mathematics and St John's College to a young, UK-based mathematics researcher. Jonathan Sherratt, who studied mathematics in Cambridge as an undergraduate, was honoured for his major contributions to Mathematical Biology.

John D. Barrow, 2006 Templeton Prize laureate, at the Templeton Prize news conference, Church Center for the United Nations, New York, March 15, 2006.
(Photo: Karen Marshall)

Congratulations to all three prize winners!

Congratulations also to Professor Peter Wadhams (Professor of Ocean Physics in DAMTP), who has been elected Foreign Member of the Finnish Academy of Science and Letters, to Professor Thanasis Fokas (Professor of Nonlinear Mathematical Science in DAMTP) who has recently been awarded an Honorary Degree from the University of Athens, and to Professor Fernando Quevedo (Professor of Theoretical Physics in DAMTP), who was awarded an Honorary Degree from Universidade del Valle de Guatemala.

We are pleased to report that a grant of £2.3m has been awarded by the Engineering and Physical Sciences Research Council Science & Innovation Awards to develop the Cambridge Statistics Initiative. The program will be based at the Centre for Mathematical Sciences and the Department of Engineering, and aims to create a centre of excellence in statistical research and teaching. Its first step will be the establishment of a new Professorship of Statistics.

To discuss any aspect of making a donation in support of mathematics at Cambridge, please contact Patrick Hawke-Smith (ph250@cam.ac.uk)

Asymptopia

CENTRE FOR MATHEMATICAL SCIENCES NEWSLETTER

UNIVERSITY OF CAMBRIDGE

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Royal recognition

By Rachel Thomas

The Millennium Mathematics Project, an education initiative based at the CMS, has won the Queen's Anniversary Prize for Higher and Further Education. The award is the highest national honour for achievement in the higher and further education sector.

The Queen's Anniversary Prize is awarded biennially to UK universities and colleges for work of exceptional quality and of broad benefit either nationally or internationally. The award was warmly welcomed by the Millennium Mathematics Project (MMP) and the University of Cambridge. John Barrow, the Director of the MMP, said: "I am delighted that the Millennium Mathematics Project has been awarded the Queen's Anniversary Prize. This is a tribute to the vision of those in the University who initiated this project, those in the outside world who added their support for it, and all the members of our dedicated project team who have made such a wide-ranging impact in schools and amongst the general public. This prize is also a welcome confirmation of the vital importance of mathematics to the United Kingdom."

The MMP aims to support maths education in schools throughout the UK through enrichment activities that promote the development of mathematical skills and understanding, and to increase the mathematical awareness and understanding of the general public. The MMP was launched in 1999 as a joint project between the Faculties of Mathematics and Education at the University of Cambridge. It includes a number of complementary programmes: *Plus* Magazine, a free online maths magazine aimed at the general public; NRICH, a website which publishes free mathematical enrichment and problem-solving material for ages 5 to 19; and Motivate, a live video-conferencing project, linking leading mathematicians, physicists and engineers to primary and secondary schools. The MMP works face to face with schools across the UK, running a Hands-On Maths Roadshow, Enigma Schools Project code breaking days, pupil workshops and continuing professional development courses and seminars for teachers. It also runs a public lecture programme aimed at schools and the general public.

Uniquely in the field of education, the anniversary prizes sit within the national honours system. The prizes originated as part of the commemorations for the fortieth anniversary of the Queen's accession to the throne. The Awards Council look in particular for "initiative, innovation and originality" that benefit the wider community. The MMP's activities have a significant regional, national and international impact, and MMP resources have been repeatedly commended by the Department for Education and Skills. Individual projects have also received national and international recognition, including an Educational Provider of the Year Award for NRICH, and a Webby – the Oscars of the internet – for *Plus*.

Professor John Barrow, director of the MMP, and Professor Alison Richard, the Vice-Chancellor of Cambridge University, receive the Queen's Anniversary Prize from Her Majesty the Queen and the Duke of Edinburgh.

The formal presentation of the award by Her Majesty the Queen took place at Buckingham Palace 16 February 2006. This is the third Queen's Anniversary Prize to be awarded to Cambridge: prizes have previously been awarded in 2002 to the Charles Darwin Correspondence Project and in 1998 to the Isaac Newton Institute for Mathematical Sciences.

Friends and Strangers

By Imre Leader

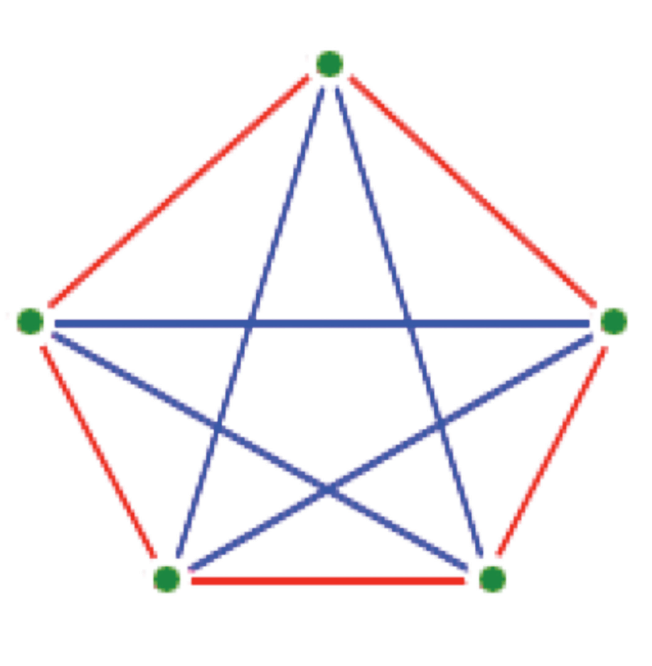
In 1928, Frank Ramsey was wrestling with a problem in mathematical logic. To solve it, he needed to show that the mathematical systems he was studying would always have a certain amount of order in them. At first sight, the systems were free to be as disorderly as they liked, but Ramsey thought that even in the most unruly, the sheer size of the system should force parts of it to exhibit some kind of order.

In proving that his intuition was correct he invented a new branch of mathematics, which is now known as Ramsey Theory. He read the resulting paper to the London Mathematical Society, but died, at the age of 26, before it was published in their *Proceedings*.

The fundamental kind of question Ramsey theory asks is: can one always find order in systems that are disordered? If so, just how large does a system have to be to contain a certain amount of order?

For a concrete example, imagine randomly selecting a group of n people. We call two people *friends* if they know each other and *strangers* if they don't. Since you've chosen the people at random, you'll probably end up with a disordered jumble of friends and strangers. To bring at least some order into this mess, you can ask the question: can I be sure that there is *either* a group of a people who are all friends, or a group of a people who are all strangers?

The answer depends on the numbers a and n. With only 5 people you cannot guarantee that there is a group of at least three friends or a group of at least three strangers, as the following graph shows:



The graph represents the 5 people as vertices, connected by a blue edge if they are friends and by a red edge if they are strangers. The graph contains no triangle that is all blue or all red, so there is no group of three friends or three strangers.

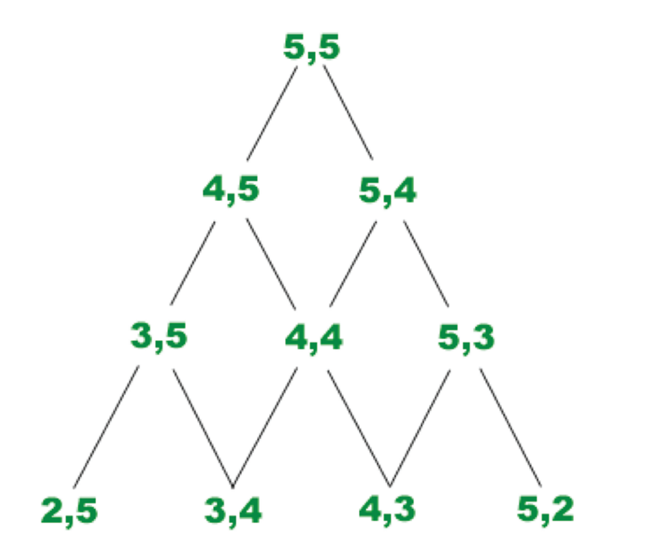
The minimum number of people necessary to guarantee either a group of a friends or a group of a strangers is called the *Ramsey number* $R(a,a)$. Similarly, we can consider $R(a,b)$, the minimum number necessary to guarantee either a group of a friends or a group of b strangers.

So does $R(a,b)$ exist for all choices of a and b? As Ramsey proved, the answer is "yes", and the proof is short and elegant.

First, note that R(2,b) and R(a,2) both exist for all values of a and b – they are equal to b and a respectively. Now, assume that for given values of a and b you know that $R(a-1,b)$ and $R(a,b-1)$ both exist and that M is the greater of the two numbers. Take a set of $2M$ people and pick an individual, call her Alice. Either Alice has at least M friends among the $2M-1$ remaining people, or she has less than M friends, and therefore at least M people are strangers to her. We assume that the former is the case; the proof will work similarly if the latter is true.

Among Alice's M friends, there will either be a group of $a-1$ who know each other, or a group of b strangers. This is because $R(a-1,b)$ is at most M. So, altogether, we have either a friends (the $a-1$ people who all know each other, together with Alice who is their friend), or we have a group of b strangers. This proves that $R(a,b)$ is at most $2M$.

Thus, working up from $R(a,2)$ and $R(2,b)$, we can prove that every Ramsey number exists. This is **Ramsey's Theorem**, and it tells us that however much orderliness we want, we can find it as long as the group of people we are given is big enough.



Proving the existence of R(5,5) breaks down into a series of steps involving smaller Ramsey numbers.

But what is the exact value of a Ramsey number $R(a,b)$? So far, we have only proved their existence. The answer is that nobody knows! The only known Ramsey numbers are $R(3,3)=6$, $R(4,3)=9$, $R(4,4)=18$, $R(4,5)=25$, $R(5,3)=14$, $R(6,3)=18$, and $R(7,3)=23$. For other values, the known bounds are not great. For example, all that is known about $R(5,5)$ is that it is between 42 and 49 inclusive.

How can it be so difficult to get an accurate value? Part of the problem is that this involves finding "counterexamples"; graphs like the one we saw representing 5 people, which show that a certain number of people is too small for given a and b. Since we are looking for examples of order, the best counterexamples will usually have lots of *disorder* – they will look random. This makes it hard or impossible to find a "rule" that gives good counterexamples. Anything constructed by a rule will probably have too much order in it.

Also, our upper bounds may be too high, but how will we ever prove it? Perhaps by examining all the possibilities on a computer? Well, suppose we wanted to show that $R(5,5)$ is at most 48. Then we would in principle have to check each graph on 48 points, and unfortunately there are 2^{1128} of these. This number is far bigger than the number of particles in the known Universe (about 10^{80}). So there is no chance of even the fastest computer imaginable ever finishing such a search. This is a puzzle we may never know the answer to.

Imre Leader is Professor of Pure Mathematics at DPMMS.

Vaccination works

By Marianne Freiberger

Infectious diseases are on everyone's mind at the moment. Bird flu has finally arrived in the UK and stringent measures to control the spread of the disease are essential. But what are the best measures? Culling all animals that are infected or at risk? This means death for thousands of healthy animals at a huge cost to farmers and the state that compensates them. Then there is vaccination, but often vaccines are much less than 100% effective, there is only a limited supply of them, and vaccinating all animals may be logistically impossible. If there is not enough vaccine for all animals on a given day, then which animals should be chosen for vaccination?

A surprisingly simple answer to this question was found by Professor Steve Brooks and his team at the Statistical Laboratory, who used a mathematical model to predict how infectious diseases spread. Using data from the 2001 foot-and-mouth epidemic in the UK, they investigated strategies that involve *responsive vaccination* – where animals close to infected farms are vaccinated in the hope that they will remain healthy. Their model allows for realistic scenarios where it is logistically and economically unfeasible to vaccinate all the animals that are at risk. The results of their study were recently published in the journal *Nature*.

According to the new method, authorities should on any given day list all the farms in order of their distance to the nearest infected farms, starting with the shortest distance. Then they should vaccinate farms in that order, using the vaccine to full capacity until it runs out. The researchers' model predicts that this prioritisation works better than any other they investigated. The good thing is that the method is very easy to implement and doesn't require any sophisticated calculations that identify which animals should be classed as "at-risk". Moreover, no change of vaccination policy is necessary even if logistical constraints change, for example if the amount of available vaccine decreases.

To model the spread of the disease the researchers constructed a probability distribution that is based on parameters such as the proximity of a farm to the closest infected farm together with epidemiological information. The values of the epidemiological parameters were estimated using data from the 2001 foot-and-mouth epidemic.

Their model predicted that vaccination in conjunction with some culling does work and that less culling is needed when vaccination is used. The responsive vaccination method described above proved to be very effective and the model also highlighted the fact that a speedy response is extremely important. "The key is to vaccinate and to vaccinate fast," says Steve Brooks.

Although the study focused on foot-and-mouth disease, its results can be applied equally well to other infectious diseases like bird flu. If the authorities learn from the study, it may help prevent the looming human pandemic. At the very least, it may in future spare us from the horrible images of thousands of burning animal carcasses that flickered across our TV screens during the 2001 foot-and-mouth epidemic.