UNIVERSITY OF CAMBRIDGE

Faculty of Mathematics

SCHEDULES FOR THE MATHEMATICS COURSES

IN THE NATURAL SCIENCES TRIPOS 2024-25

This booklet describes courses provided by the Faculty of Mathematics for students taking Natural Sciences or Computer Science. Its purposes are to provide :

(i) detailed schedules (i.e. syllabuses) for each of the courses;

(ii) information about the examinations;

(iii) a bibliography.

Queries and suggestions should be addressed to :

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All the relevant documentation is available on the Faculty of Mathematics website (`http://www.maths.cam.ac.uk/`), where a section specific to the Mathematics courses in the Natural Sciences Tripos (`http://www.maths.cam.ac.uk/undergradnst/`) is provided. Course materials are available on Moodle at `https://www.vle.cam.ac.uk/`.

November 22, 2024

AIMS AND OBJECTIVES

The aim of the Faculty of Mathematics is to provide relevant service courses for the Natural Sciences and Computer Science Triposes. After completing Mathematics A or B (Part IA) and Mathematics (Part IB), students should have covered the mathematical methods required to provide a grounding in the mathematical techniques used either in the Physical Sciences courses of the Natural Sciences Tripos or in the Computer Science Tripos, as appropriate.

COURSES

Part IA

The following mathematics courses are provided for Part IA of the Natural Sciences and Computer Science Tripos.

- Mathematics, Course A
- Mathematics, Course B

Course A provides a thorough grounding in methods of mathematical science and contains everything prerequisite for the mathematical content of all physical-science courses in Part IB of the Natural Sciences Tripos, including specifically Mathematics, Physics A and Physics B. Course B contains additional material for those students who find mathematics rewarding in its own right, and it proceeds at a significantly faster pace. Both courses draw on examples from the physical sciences but provide a general mathematical framework by which quantitative ideas can be transferred across disciplines.

Students are strongly encouraged to take Course A unless they have a thorough understanding of the material in Further Mathematics A-Level. Some topics that look similar in the Schedules may be lectured quite differently in terms of style and depth. Both courses lead to the same examination and qualification. **Mathematics is a skill that requires firm foundations: it is a better preparation for future courses in NST to gain a first-class result having pursued Course A than to gain a second-class result following Course B.**

Each course consists of 60 lectures over three terms.

An additional scientific computing module organised by the NST Management Committee runs in Michaelmas Term; information regarding this module will be provided separately.

Part IB

The following mathematics course is provided for Part IB of the Natural Sciences Tripos

- Mathematics

In order to take this course in Part IB of the Natural Sciences Tripos, it is recommended to have obtained at least a second class in Part IA Mathematics, course A or B. The material from course A is assumed. Students are nevertheless advised that if they have taken course A in Part IA, they should consult their Director of Studies about suitable reading during the Long Vacation before embarking upon part IB Mathematics. The IB course consists of 58 lectures over the three terms, six assessed computational exercises and occasional examples classes.

EXAMINATIONS

In the examinations, formulae booklets will not be provided but candidates will not be required to quote elaborate formulae from memory. The use of calculators will not be permitted.

Part IA Mathematics, courses A and B

These courses are examined in two three-hour written papers, common to both courses, at the end of the year.
The written papers each consist of two sections, A and B. Section A on Paper 1 is based on the A-Level syllabus. All other parts of the written papers are based on these Schedules. Candidates may attempt all questions from Section A and at most 5 questions from section B.
Section A on each paper consists of up to 20 short-answer questions and carries a total of 20 marks. Section B on each paper consists of 10 questions, each of which carries 20 marks. Up to 2 of the questions in Section B of each paper are starred to indicate that they rely on material lectured in the B course but not in the A course. The examination paper shows, for each major subsection of a question, the approximate maximum mark available.
The questions in Section A have clear goals that carry 1 mark (correct) or 0 marks (incorrect or incomplete); no fractional credit is given and it is not necessary to show working. In Section B, partial credit may be available for incomplete answers and students are advised to show their working.

Part IB Mathematics

This course is examined in two three-hour papers at the end of the year, together with six assessed computer practicals. The arrangements for the practicals are described in a course handbook that can be found on Moodle.
On each paper, candidates may attempt up to 6 questions. All attempts at a question are given a mark out of 20. The examination paper shows, for each major subsection of the question, the approximate maximum mark available.
The total credit available for the computer practicals is 24 marks: each of the six modules has a maximum mark of 4. Students are required to register electronically for the Computer Practical course using the instructions in the course handbook. This must be done before the deadline (in late October) stated in the handbook. The first three modules must be submitted in electronic form before the end of the Michaelmas term and the three remaining modules must be submitted for assessment in electronic form by the end of the Lent term. The student must also submit a signed declaration form stating that 'The results achieved are my own unaided work'.

Transcript Marks

As with all mathematics examinations, the marks required for each class vary from year to year according to the difficulty of the examination and other factors.

In order to ensure comparability between subjects, the Natural Sciences Tripos requires marks to be reported in a form where $70+\%$ corresponds to a First Class, $60-69\%$ to a 2.i, $50-59\%$ to a 2.ii, and $40-49\%$ to a Third. In accordance with guidelines for the NST, this reported mark, which appears on CamSIS and official transcripts, is obtained by a piecewise linear mapping of the total raw marks onto this scale, and the same mapping is applied, in proportion, to the raw marks obtained on both the examinations and the practicals.

SCHEDULES

The schedules, or syllabuses, given on the following pages are determined by a committee which has input from all the Physical Science subjects in the Natural Sciences and from Computer Science and is agreed by the Faculty of Mathematics. The schedules are minimal for lecturing and maximal for examining; that is to say, all the material in the schedules will be lectured and only this material will be examined.

The numbers in square bracket at the end of paragraphs of the schedules indicate roughly the number of lectures that will be devoted to the material in the paragraph. Topics marked with asterisks should be lectured, but questions will not be set on them in examinations.

Part IA: Mathematics, course A

This course comprises Mathematical Methods I, Mathematical Methods II and Mathematical Methods III.

The material in the course will be as well illustrated as time allows with examples and applications of Mathematical Methods to the Physical Sciences.

Mathematical Methods I **24 lectures, Michaelmas term**

Vector sum and vector equation of a line. Scalar product, unit vectors, vector equation of a plane. Vector product, vector area, vector and scalar triple products. Orthogonal bases. Cartesian components. Spherical and cylindrical polar coordinates. [**5**]

Complex numbers and complex plane, vector diagrams. Exponential function of a complex variable. $\exp(\mathrm{i}\omega t)$, complex representations of cos and sin. Hyperbolic functions. [**3**]

Revision for functions of a single variable of differentiation (including differentiation from first principles, product and chain rules) and of stationary values. Elementary curve sketching. Brief mention of the ellipse and its properties. Power series. Statement of Taylor's theorem. Examples to include the binomial expansion, exponential and trigonometric functions, and logarithm. Newton-Raphson method. [**5**]

The integral as the limit of a sum. Methods of integration (including by parts and substitution). Examples to include odd and even functions and trigonometric functions. Fundamental theorem of calculus. [**3**]

Elementary probability theory. Simple examples of conditional probability. Probability distributions, discrete and continuous, normalisation. Permutations and combinations. Binomial distribution, $(p+q)^n$, binomial coefficients. Normal distribution. Expectation values, mean, variance and its expression in terms of first and second moments. [**5**]

Extended examples distributed through the course. [**3**]

Mathematical Methods II **24 lectures, Lent term**

Ordinary differential equations. First order equations: separable equations; linear equations, integrating factors. Second-order linear equations with constant coefficients; $\exp(\lambda x)$ as trial solution, including degenerate case. Superposition. Particular integrals and complementary functions. Constants of integration and number of necessary boundary/initial conditions. Particular integrals by trial solutions. [**6**]

Differentiation of functions of several variables. Differentials, chain rule. Exact differentials. Scalar and vector fields. Gradient of a scalar as a vector field. Directional derivatives. Unconditional stationary values. Elementary sketching of contours in two dimensions illustrating maxima, minima and saddle points. Verification of solution to a partial differential equation by substitution. Linear superposition. [**7**]

Double and triple integrals in Cartesian, spherical and cylindrical coordinates. Examples to include evaluation of $\int_{-\infty}^{+\infty} \exp(-x^2)\mathrm{d}x$. [**3**]

Parameterized curves. Line integral of a vector field. Conservative and non-conservative vector fields. Surface integrals and flux of a vector field over a surface. Divergence of a vector field. ∇^2 as div grad. Curl. Statements (only) of the divergence and Stokes theorems. [**5**]

Extended examples distributed through the course. [**3**]

Mathematical Methods III **12 lectures, Easter term**

Linear equations. Matrix addition and multiplication. Determinant of a matrix. Statement of main properties of determinants. Inverse matrix. The equations $\mathbf{A}\,\mathbf{x} = \mathbf{0}$ with non-zero solutions. Symmetric, antisymmetric and orthogonal matrices. Eigenvalues and eigenvectors for symmetric matrices. [**7**]

Orthogonality relations for sine and cosine. Fourier series; examples. [**3**]

Extended examples distributed through the course. [**2**]

Part IA: Mathematics, course B

This course comprises Mathematical Methods I, Mathematical Methods II and Mathematical Methods III.

The material will be as well illustrated as time allows with examples and applications of Mathematical Methods to the Physical Sciences.

Mathematical Methods I **24 lectures, Michaelmas term**

Vector sum and vector equation of a line. Scalar product, unit vectors, vector equation of a plane. Vector product, vector area, vector and scalar triple products. Orthogonal bases. Cartesian components. Spherical and cylindrical polar coordinates. [**4**]

Complex numbers and complex plane, vector diagrams. Exponential function of a complex variable. $\exp(\mathrm{i}\omega t)$, complex representations of cos and sin. Hyperbolic functions. [**2**]

Revision of single variable calculus. Leibnitz's formula. Elementary curve sketching. Brief mention of the ellipse and its properties. Elementary Analysis; idea of convergence and limits. Orders of magnitude and approximate behaviour for large and small x. O notation. Idea of continuity and differentiability of functions. Power series. Statement of Taylor's theorem. Examples to include binomial expansion, exponential and trigonometric functions, and logarithm. Newton-Raphson method. Convergence of series; comparison and ratio tests. [**6**]

The integral as the limit of a sum. Differentiation of an integral with respect to its limits or a parameter. Approximation of a sum by an integral. Stirling's approximation as an example. Schwarz's inequality. Double and triple integrals in Cartesian, spherical and cylindrical coordinates. Examples to include evaluation of $\int_{-\infty}^{+\infty} \exp(-x^2)\,\mathrm{d}x$. [**5**]

Elementary probability theory. Simple examples of conditional probability. Probability distributions, discrete and continuous, normalisation. Permutations and combinations. Binomial distribution, $(p+q)^n$, binomial coefficients. Normal distribution. Expectation values, mean, variance and its expression in terms of first and second moments. [**4**]

Extended examples distributed through the course. [**3**]

Mathematical Methods II **24 lectures, Lent term**

Ordinary differential equations. First order equations: separable equations; linear equations, integrating factors. Examples involving substitution. Second-order linear equations with constant coefficients; $\exp(\lambda x)$ as trial solution, including degenerate case. Superposition. Particular integrals and complementary functions. Constants of integration and number of necessary boundary/initial conditions. Particular integrals by trial solutions. Examples including radioactive sequences. Resonance, transients and damping. [**6**]

Differentiation of functions of several variables. Differentials, chain rule. Exact differentials, illustrations including Maxwell's relations. Scalar and vector fields. Gradient of a scalar as a vector field. Directional derivatives. Unconditional stationary values; classification using Hessian matrix. Conditional stationary values, Lagrange multipliers, examples with two or three variables. Boltzmann distribution as an example. [**8**]

Parameterized curves. Line integral of a vector field. Conservative and non-conservative vector fields. Surface integrals and flux of a vector field over a surface. Divergence of a vector field. ∇^2 as div grad. Curl. Divergence and Stokes's theorems. [**5**]

Orthogonality relations for sine and cosine. Fourier series; examples. [**2**]

Extended examples distributed through the course. [**3**]

Mathematical Methods III **12 lectures, Easter term**

Linear equations. Notion of a vector space; linear mappings. Matrix addition and multiplication. Determinant of a matrix. Statement of the main properties of determinants. Inverse matrix. Equations $\mathbf{A}\,\mathbf{x} = \mathbf{0}$ with non-zero solutions. Symmetric, antisymmetric and orthogonal matrices. Eigenvalues and eigenvectors for symmetric matrices. Hessian matrix as an example. [**6**]

Linear second-order partial differential equations; physical examples of occurrence, verification of solution by substitution. Linear superposition. Method of separation of variables (Cartesian coordinates only). [**4**]

Extended examples distributed through the course. [**2**]

Part IB: Mathematics

This course comprises Mathematical Methods I, Mathematical Methods II, Mathematical Methods III and six Computer Practicals. The material in Course A from Part IA will be assumed in the lectures for this course.

The material in the course will be as well illustrated as time allows with examples and applications of Mathematical Methods to the Physical Sciences. Separate occasional examples classes will be given as stated in the lecture list.

Mathematical Methods I — 24 lectures, Michaelmas term

Fourier transform
Fourier transforms; relation to Fourier series, simple properties and examples, convolution theorem, correlation functions, Parseval's theorem and power spectra; brief introduction of the Laplace transform. [**2**]

Vector calculus
Suffix notation. Einstein summation convention. Contractions using δ_{ij} and ϵ_{ijk}. Reminder of vector products, grad, div, curl, ∇^2, and their representations using suffix notation. Divergence theorem and Stokes' theorem. Vector differential operators in orthogonal curvilinear coordinates, e.g. cylindrical and spherical polar coordinates. Jacobians. [**6**]

Partial differential equations
Linear second-order partial differential equations; physical examples of occurrence, the method of separation of variables (Cartesian coordinates only). [**2**]

Green's functions
Response to impulses, delta function (treated heuristically), Green's functions for initial and boundary value problems. [**3**]

Matrices
N–dimensional vector spaces, matrices, scalar product, transformation of basis vectors. Eigenvalues and eigenvectors of a matrix; degenerate case, stationary property of eigenvalues. Orthogonal and unitary transformations. Quadratic and Hermitian forms, quadric surfaces. [**5**]

Elementary Analysis
Idea of convergence and limits. O notation. Statement of Taylor's theorem with discussion of remainder. Convergence of series; comparison and ratio tests. Power series of a complex variable; circle of convergence. Analytic functions: Cauchy-Riemann equations, rational functions and $\exp(z)$. Zeros, poles and essential singularities. [**3**]

Series solutions of ordinary differential equations
Homogeneous equations; solution by series (without full discussion of logarithmic singularities), exemplified by Legendre's equation. Classification of singular points. Indicial equation and local behaviour of solutions near singular points. [**3**]

Computer practicals **Michaelmas & Lent terms**

There are no lectures for this course, which consists of six computational exercises related to material elsewhere in the Mathematics course.

Topics for the exercises will include :

1. Familiarisation, getting started. Numerical integration.
2. Solving ordinary differential equations.
3. Root finding.
4. Solving partial differential equations.
5. Matrix algebra.
6. Eigenfunction expansions.

Mathematical Methods II **24 lectures, Lent term**

Sturm-Liouville theory
Self-adjoint operators, eigenfunctions and eigenvalues, reality of eigenvalues and orthogonality of eigenfunctions. Eigenfunction expansions and determination of coefficients. Legendre polynomials; orthogonality. [**3**]

Conditional stationary values and the calculus of variations
Lagrange multipliers, examples with two or three variables. Euler-Lagrange equations and examples.

Variational principles; Fermat's principle; Hamilton's principle and deduction of Lagrange's equation, illustrated by a system with:

$$L = \tfrac{1}{2}\, m_1 \dot{x}_1^2 + \tfrac{1}{2}\, m_2 \dot{x}_2^2 - V(x_1 - x_2)\,.$$

Variational principle for the lowest eigenvalue *and for higher eigenvalues* (Rayleigh-Ritz). [**6**]

Laplace and Poisson's equations
Solution by separation of variables of Laplace's equation in plane polar coordinates, and spherical polar coordinates (axisymmetric case); Legendre polynomials again.

Solution of Poisson's equation as an integral. Uniqueness for Poisson's equation with Dirichlet boundary conditions. Green's identity. Green's function for Laplace's equation with simple boundary conditions using the method of images. Applications to electrostatic fields and steady heat flow. [**5**]

Cartesian tensors
Transformation laws, addition, multiplication, contraction. Isotropic tensors, symmetric and anti-symmetric tensors. Principal axes and diagonalisation. Tensor fields, e.g. conductivity, polarizability, elasticity. [**4**]

Contour integration
Integration along a path; elementary properties. Cauchy's theorem; proof by Cauchy-Riemann equations and divergence theorem in 2–D. Integral of $f'(z)$; Cauchy's formula for $f(z)$. Calculus of residues; examples of contour integration; point at infinity; multi-valued functions, branch points, log (z). [**4**]

Transform methods
Fourier inversion by contour integration. Examples of simple linear differential equations, including diffusion equation. [**2**]

Mathematical Methods III **10 lectures, Easter term**

Small oscillations

Small oscillations and equilibrium; normal modes, normal coordinates, examples, e.g. vibrations of linear molecules such as CO_2. Symmetries of normal modes. [**2**]

Group theory

Idea of an algebra of symmetry operations; symmetry operations on a square. Definition of a group; group table. Subgroups; homomorphic and isomorphic groups.

Representation of groups; reducible and irreducible representations; basic theorems of representation theory. Classes, characters. Examples of character tables of point groups. *Applications in Molecular Physics*. [**8**]

BIBLIOGRAPHY

There are very many books which cover the sort of mathematics required by Natural Scientists. The following should be helpful as general reference; further advice will be given by Lecturers. Books which can reasonably be used as principal texts for the course are marked with a dagger. The prices given are intended as a guide only, and are subject to change.

Natural Sciences Mathematics Part IA

† M L Boas
Mathematical Methods in the Physical Sciences, 2nd edition.
Wiley, 1983 (3rd edition available August 2005).

A Jeffrey
Mathematics for Engineers and Scientists, 5th edition.
Nelson Thornes, 1996 (6th edition available)

† E Kreyszig
Advanced Engineering Mathematics, 8th edition.
Wiley, 2011 (10th edition available).

† K F Riley, M P Hobson & S J Bence
Mathematical Methods for Physics and Engineering.
3rd ed., Cambridge University Press, 2002
(Available online via `http://idiscover.lib.cam.ac.uk`).

I S Sokolnikoff & R M Redheffer
Mathematics of Physics and Modern Engineering.
McGraw Hill, 1967 (2nd edition available)

† G Stephenson
Mathematical Methods for Science Students, 2nd edition.
Prentice Hall/Pearson, 1973 (Dover Books on Mathematics).

G Stephenson
Worked Examples in Mathematics for Scientists and Engineers.
Part of: Dover Books on Mathematics ($ 15.89)

K A Stroud & D Booth
Engineering Mathematics, 5th edition.
Palgrave, 2001 (6th edition available)

K A Stroud & D Booth
Advanced Engineering Mathematics.
Palgrave, 2003 (6th edition available)

G Thomas, M Weir & J Hass
Thomas's Calculus, 11th edition.
Pearson, 2014 (14th edition available)

Natural Sciences Mathematics Part IB

† G Arfken & H Weber
Mathematical Methods for Physicists, 6th edition.
Elsevier, 2005 (Available as hardcover or paperback).

† J W Dettman
Mathematical Methods in Physics and Engineering.
Dover, 1988 (Dover Books on Physics).

† H F Jones
Groups, Representation and Physics, 2nd edition.
Institute of Physics Publishing (Taylor & Francis), 1998
(Available as hardcover or paperback).

E Kreyszig
Advanced Engineering Mathematics, 8th edition.
Wiley, 1999 (10th edition available)

J Mathews & R L Walker
Mathematical Methods of Physics, 2nd edition.
Pearson/Benjamin Cummings, 1970 (Available as hardcover or paperback)

† K F Riley, M P Hobson & S J Bence
Mathematical Methods for Physics and Engineering.
3rd ed., Cambridge University Press, 2002 (Available as hardcover or paperback).

R N Snieder
A guided tour of mathematical methods for the physical sciences, 2nd edition.
Cambridge University Press, 2004 (3rd edition available)