UNIVERSITY OF CAMBRIDGE

Faculty of Mathematics

STUDY SKILLS IN MATHEMATICS

Handout for first–year students.

All the documentation is available on the WWW (http://www.maths.cam.ac.uk/).

September 16, 2011

1 Introduction

Learning mathematics is very different from learning other subjects, as anyone who tries to read a mathematics text book soon finds out. And learning mathematics at university is very different from learning mathematics at school, as anyone who sits through a university lecture soon finds out. *Intense* is the word that springs to my mind when I try to describe the difference in style between school and university mathematics.

Lectures are intense compared with lessons because there are comparatively few of them; supervisions are intense compared with lessons because you go over a week's work in a single hour; work is intense because terms are much shorter (the 8 weeks looming ahead of you may seem an eternity, but I promise that in 8 weeks time you will be wondering what happened to the time); and examinations are intense because you have to cram a year's work into 4 three-hour papers taken in the space of a few days.

This booklet is intended specifically for first year mathematicians; it was written for you. I hope that it will help you to make the most of your time at Cambridge.

There is also a booklet that is specifically intended for supervisors. You may well find it interesting and even useful to read the advice we give them on how to supervise. The pdf version is on the faculty web site:
`www.maths.cam.ac.uk/facultyoffice/supervisorsguide`.

Stephen Siklos,
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2 Lectures

The really big difference between school and university is in lectures. The lecturers have only 12 lectures a week in which to give you enough material to keep you occupied for the other 156 hours. Therefore, the material comes at you pretty fast.

It follows that, whereas at school you probably expected to understand what the teacher said as it was said, here there will be great chunks of the notes which you will not understand until you have worked on them later: line by line, if necessary. Even then, there may be some parts of the course that only really come clear when you come to revise the material.[1]

Nevertheless, it is important to try to understand as much as possible of what is being said as it is said. Apart from saving time later, you may otherwise miss vital explanations and insights. So:

- Do make the effort to concentrate.[2] We have all heard that, in a mathematics lecture, what the lecturer writes on the blackboard goes straight into the student's notebook without passing through the brain of either. You should do everything in your power to prevent this happening: sit near the front if you find 50 minutes of mathematics a strain; don't let your thoughts wander; and remember that concentration is just a matter of self-discipline and practice.

- Do ask questions during the lecture rather than let something pass by. If the lecturer is writing too fast, or too illegibly, or is speaking too quietly for you, it is likely that others are having the same difficulty.

- Don't be afraid to ask what you may think is a silly question. Nine times out of ten, most of the rest of the audience will be impressed (if only with your bravery) and many of them will

[1] Reread this last paragraph! It is easy to become discouraged if you do not fully appreciate this message.

[2] *The brain is a wonderful organ; it starts working the moment you get up in the morning and does not stop until you get into the lecture theatre.* Robert Frost (American poet 1874 – 1963).

also want to know the answer. And it is just possible that the lecturer has made a mistake.

- Do try to appear responsive: look up when you have finished writing and are ready for more (this helps the lecturer pace the lecture); look puzzled when you are puzzled (so that the lecturer knows when more explanation is required); allow a gleam of recognition to surface if you suddenly realise what is going on. (Bob Hope, the American comedian, used to say that he liked British audiences, because even if they don't feel like laughing at one of his jokes, they nod their heads to show that they have understood it.)

The usual convention in lectures is that you write down exactly what the lecturer writes on the board or OHP. You may find that you can supplement this a little during the lecture but often there will not be time. Here are some very important trivialities:

- Write the page number and lecture number on each sheet; if you drop your notes or get into a muddle photocopying them, you will find that one page of mathematics looks very like any other. This may seem a ludicrously trivial point. A few years ago when there was a severe water shortage the water boards put out an advertisement telling everyone to turn off the tap while brushing teeth. The purpose was not just to save the cupful of water, but to put people in the right frame of mind for making other more significant savings. You will find that numbering your pages will help with the overall orderliness that can be a great time saver.
- Leave wide margins; you will certainly need to annotate when you go through later.
- File your work (annotated lecture notes and supervision material) in an orderly way; this will save a lot of time when you come to revise.

Here is the most important tip: **you will save an immense amount of time if you always get to grips with one lecture before going to the next**. This way, you will get much more out of the lectures, which will in turn save time when you go through your notes later. You should therefore set aside a slot each day for going through your lecture notes — not just reading them, but working through them line by line. This is easy to say but hard to do; as soon as you fall behind it requires an enormous effort to catch up again.

> One final point. You may think that the lecturer is talking to you as a big group, but the lecturer actually sees a large number of individuals. You should extend to the lecturer the normal courtesies of an individual conversation; behave as if the lecturer is talking to you personally. Don't, for example, spend the lecture chatting to your neighbour or reading the newspaper. This is most distracting for the rest of the audience and also for the lecturer, and is a sure recipe for a poor lecture. And **please** remember to turn off your mobile phone in lectures; or, better still, leave it in your room.

3 Lecturing styles

You will find that lecturers adopt a range of strategies for conveying to you the material listed in the schedules.

For example, some lecturers work entirely on the blackboard or on overhead projectors; some give out a complete set of printed notes; some cover theory (say) on the blackboard and give out the examples (say) on hand-outs. Some styles will suit you better than others, but it is very much a personal matter; do not assume that others will agree with you about what is best.

Often, students — especially first year students who are used to A-level learning methods — want complete printed lecture notes, thinking that this is what they need to *learn* the material. That may be so, but the aim is to *understand* the material, which is a very different matter. For this, it may be much more useful to have a carefully distilled set of notes; the work you do in fleshing out the details will serve you far better in the long run than reading a complete set of printed notes.

4 Supervisions

There is not much scope for heated discussion of topical issues in mathematics supervisions. In fact, any debate at all is difficult, since your opinion seems not to count for much when your supervisor knows all the right answers (having been told them by his or her supervisors when they did the course). I mentioned above that lectures are to some extent interactive; this is very much the case with supervisions. A good part of the responsibility for making the supervision useful and interesting lies with you. Remember that most supervisors are human beings too: they like you to talk to them and show an interest (e.g. by asking questions) in what they are telling you.

Generally, in a mathematics supervision, you sit at a desk with your supervisor who will write out solutions to exercises or explanations of pieces of mathematics on paper (*not* on a blackboard). You should not take notes yourself; leave your mind completely free to concentrate on understanding everything your supervisor says. At the end of the supervision, you take away what he or she has written and (best) use it to annotate, correct or complete your own supervision work or lecture notes, or (second best) file it with your own work. Your supervision partner can use the notes after you, or photocopy them.

> You **must** hand your work in well before the supervision. Your work should be neatly and clearly presented. If your work is scribbled and scruffy, then you should rewrite it. It should be logically and carefully argued otherwise it is not mathematics. You may *think* you can do a problem before you even set pen to paper, but you don't *know* that you can do a problem until you write out all the details. Also, unless you are in the habit of writing careful solutions, you will come unstuck in the examinations when you do not have the opportunity to explain what you really meant.

If you do not make good use of supervisions, then you will squander one of the most important (and expensive) assets that Cambridge has to offer. From long years of experience, I recommend that to make best use of your supervisions, you should:

- bring your lecture notes to the supervision, having marked in the bits you don't follow;
- hand your work in on time, so that it can be marked thoroughly;
- mark your own work: make a note in the margin wherever there is a step you are not sure of, or which you have missed out. I am not just telling you this to ingratiate myself with your supervisors, though it will undoubtedly make their lives easier: a self-critical approach to your own work will prove invaluable when it comes to exam time;
- tell your supervisor (if appropriate and polite) exactly what you would like him or her to do — remember that sometimes your supervisor will have much less experience of supervisions than you, and will be glad of your advice;
- make sure your supervisor writes down enough on each example for you to reconstruct the solution afterwards;
- have an intelligent question ready in case the supervision is grinding to a halt with time to spare;
- review the supervision as soon as possible afterwards, while it is still fresh in your mind.

Last of all, here is the most important tip: do not be lazy. It is very easy to let what the supervisor is saying just wash over you, perhaps hoping that all will come clear later. **If you don't understand what the supervisor has done, say so.**

5 Supervision reports

At the end of each term, each of your supervisors will lodge a report for you on CamCORS[3]. It has to be released by your Director of Studies before you can read it: this is just a precaution, to cope with the extremely unlikely event of a completely inappropriate report, or a muddling of names, or some other disaster. You will receive an automatic e-mail as soon as a supervision report is available. It is important that you read your reports, and if they don't appear to be there, ask your Director of Studies to investigate. Usually, they will appear (because supervisors are not paid until they file a report), but they might be late in which case they may arrive at a time when your Director of Studies is not expecting them and will not release them without a prod from you.

6 Guide for supervisors

If you would like to see how the process of supervision looks from the 'other side', you can see our (rather good) guide for supervisors: `http://www.maths.cam.ac.uk/facultyoffice/supervisorsguide/`
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7 Writing mathematics

Most mathematicians can write accurate grammatical prose, they understand (for example) why the comma in this sentence should have been a full stop or a semi-colon. There is a grammar to writing mathematics as well. Symbols such as $\forall$, $\Rightarrow$, $\exists$, etc, should be used in a way that makes grammatical sense if read out in full. If you are careless about this, then you will certainly find yourself using sloppy logic as well as sloppy mathematical grammar.

You should try to write in full sentences, using normal punctuation: full stop at the end of a sentence even if ends in an equation,

[3]Cambridge Colleges Online Reporting System; your Director of Studies will explain about this.

commas normally in pairs, etc. Sentences should be short and as simple as possible. If you find yourself writing paragraphs of text, then you should consider whether you are writing more than is necessary to explain what you are doing.[4]

You need to be absolutely precise in your mathematical writing.[5]

Finally, remember that you are laying out your thoughts for someone else. You mustn't think 'Well, I'm sure he or she will know what I mean'. The reader may well be able to guess at what you mean but, if the reader is your examiner, this may not get you your marks. In any case, why make your reader do the work?

8 Solving problems

Mathematics is all about problem solving and the only way to test your understanding of the material is to work through examples. At school, problems were fairly short and the answers came out neatly. As an undergraduate, you will find that many problems take ages to do; even if you know exactly what you are doing, each problem may take a considerable time and several sheets of A4 to complete. (This is as it should be: at research level, problems take months or years or may simply be unsolvable.) Here are some thoughts on tackling problems.

If you cannot get started on a problem, try the following, in order.

- Reread the question to check that you understand what is wanted.
- Reread the question to look for clues – the way it is phrased, or the way a formula is written, or other relevant parts of the question. (You may think that the setters are trying to set difficult questions or to catch you out; nothing could be further from the truth. They are probably doing all in their power to make it easy for you by trying to tell you what to do).

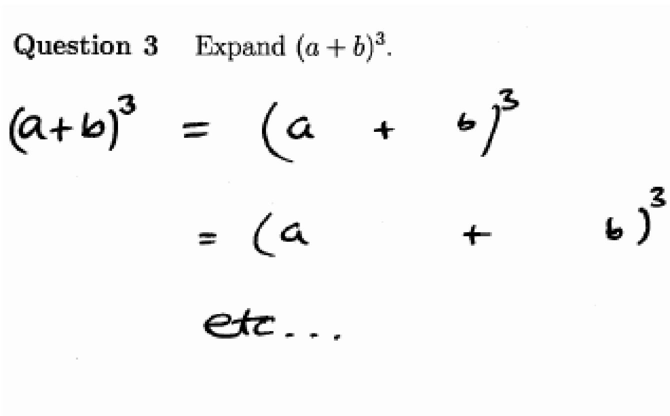
[4]Note: in this sentence, if is followed by then; it sounds a bit stilted here but you should always adhere to this rule in a mathematical sentence.

[5]Kevin Houston's very nice book *How to Think Like a Mathematician* (Cambridge University Press, 2009) starts with the question 'How many months have 28 days?' and then gives the mathematicians answer: 'All of them'.

- Simplify the notation – e.g. by writing out sums or vector components explicitly.

- Look at special cases (reduce the number of dimensions, choose special values which simplify the problem) to try to understand why the result is true.

- Try to understand what it is that you don't understand. For example, look up the definitions of the technical terms – often this will open up new vistas.



Question 3 Expand $(a+b)^3$.

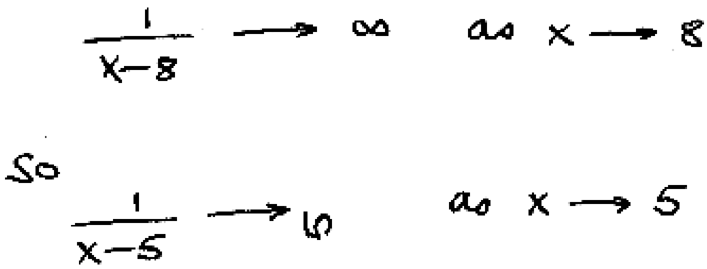
$(a+b)^3 = (a \quad + \quad b)^3$

$= (a \quad\quad + \quad\quad b)^3$

etc...

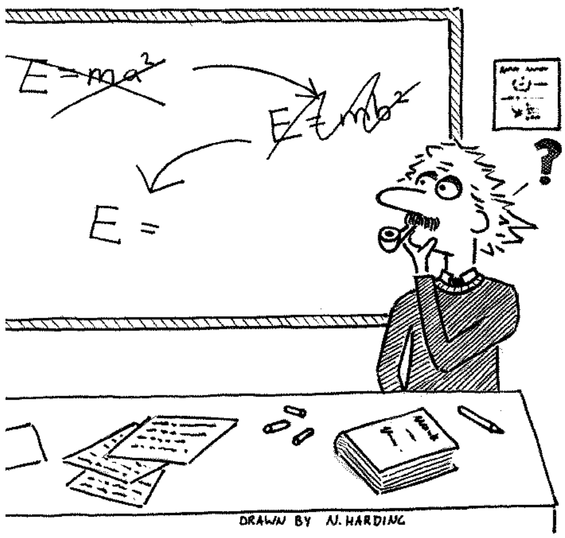
Make sure that you understand fully the technical terms used in the statement of the problem.

- Look for a similar problem in your notes or in a textbook.



But make sure that you understand fully the example you are working from.

- Write down your thoughts – in particular, try to express the exact reason why you are stuck.



Solving problems: write down your thoughts …

- Go onto the next question and go back later.

- Take a (short!) break. Littlewood (distinguished Cambridge mathematician and author of the highly entertaining *Mathematicians Miscellany* — dip into it in your college library, ignoring the last section if it is a recent edition) used to work seven days a week until an experiment revealed that when he took Sundays off the good ideas had a way of coming on Mondays.

- Ask a friend (but make sure you still think it through yourself – friends are not infallible). BUT: remember that following someone else's solution (whether by supervisor, lecturer or friend) is not remotely the same thing as doing the problem yourself.

This is important: once you have seen someone else's solution to an example, then you are deprived, for ever, of most of the benefit that could have come from trying it yourself. Even if, ultimately, you get stuck on a particular problem, you derive vastly more benefit from seeing a supervisor's solution to a problem with which you have already struggled, than by simply following a solution to a problem to which you've given very little thought.

If you have got started but the answer doesn't seem to be coming out then check your algebra. In particular, make sure that what you have written works in special cases. For example: if you have written the series for $\log(1+x)$ as

$$1 - x + x^2/2 - x^3/3 + \cdots$$

then a quick check will reveal that it doesn't work for $x = 0$; clearly, the 1 should not be there.[6]

You should also make sure that what you have written makes sense. For example, in a problem which is dimensionally consistent, you cannot add x (with dimensions length, say) to x^2 or $\exp x$ (which itself does not make sense). Even if there are no dimensions in the

[6] Another check will reveal that for very small positive x, log is positive (since its argument is bigger than 1) whereas the series is negative, so there is clearly something else wrong.

problem, it is often possible to mentally assign dimensions and hence enable a quick check.

Be wary of applying familiar processes to unfamiliar objects (very easy to do when you are feeling at sea): for example, when dealing with matrices, it is all too easy to write $\mathbf{AB}$ instead of $\mathbf{BA}$ with nice simplifications in view; or to solve the vector equation $\mathbf{a}.\mathbf{x} = 1$ by dividing both sides by $\mathbf{a}$. If your calculations seem to be all right, then go over the steps above.

If you are *not* stuck, then:

- Write out the solution fully; it is not good enough just to glance at an example and skip it if it looks easy.
- Look back over what you have done, checking that the arguments are correct and making sure that they work for any special cases you can think of. It is surprising how often a chain of completely spurious arguments and gross algebraic blunders leads to the given answer.
- Make sure that you are not unthinkingly applying mathematical tools which you do not fully understand.[7]
- Try to see how the problem fits into the wider context and see if there is a special point which it is intended to illustrate.
- Make sure that you actually understand not only what you have done, but also why you have done it that way rather than some other way. This is particularly important if you have worked

[7]Mathematicians should feel as insulted as Engineers by the following joke.

A mathematician, a physicist and an engineer enter a mathematics contest, the first task of which is to prove that all odd number are prime. The mathematician has an elegant argument: '1's a prime, 3's a prime, 5's a prime, 7's a prime. *Therefore*, by mathematical induction, all odd numbers are prime. It's the physicist's turn: '1's a prime, 3's a prime, 5's a prime, 7's a prime, 11's a prime, 13's a prime, so, to within experimental error, all odd numbers are prime.' The most straightforward proof is provided by the engineer: '1's a prime, 3's a prime, 5's a prime, 7's a prime, 9's a prime, 11's a prime . . . '.

from a similar example in the notes (or if you sought advice from a friend).

9 Examinations

Tripos's heroic period, at the end of the nineteenth century, candidates had to sit through 36 hours of extremely hard papers. One year in the 1880's, the maximum possible mark was 33,541 and the Senior Wrangler (that is the man who came top of the list) scored 16,368, i.e. about half marks, which works out at roughly 8 marks/minute. The Wooden Spoon (that is the man who came bottom of the pass list) amassed a grand total of 247.

Nowadays, the examinations are much more friendly, being designed to test your knowledge of the courses you have attended rather than your ability to jump through mathematical hoops. Nevertheless, strategy matters. Extreme marks (either high or low) are available in mathematics examinations, which means that playing the cards you hold to best advantage is of vital importance.

Here are some thoughts; you've heard them all before, but that does not make them any less worth saying. The examinations may be some way off, but you will find that good examination technique can be acquired over the course of the year by making suitable preparations and developing good habits. (For example, the first two points assume that your year's work is in good order.)

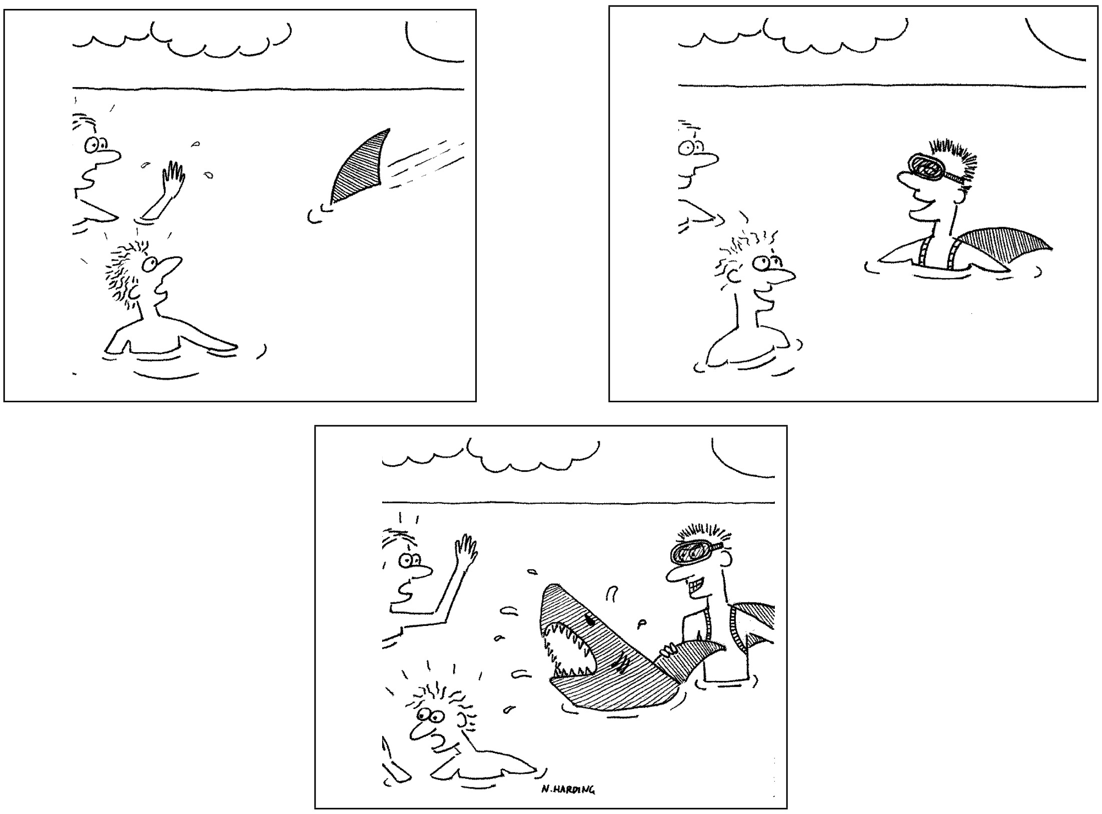
- For revision, work through examples while reading the relevant section of your notes (just reading is not enough).

- For last minute preparation, look through your supervision work to remind yourself how to do questions.

- In the examination, above all, stay cool – if it is hard for you, it is probably hard for everyone.

- Don't rush into a question – read the whole paper carefully and start with the question you feel most confident about.

- Analyze exactly what you are being asked to do; try to understand the hints (explicit and implicit); remember to distinguish between terms such as explain/prove/define/etc.
- Remember that different parts of a question are often linked (it is usually obvious from the notation) and choice of variables.
- Set out your answer legibly and logically (don't scribble down the first thought that comes into your head) – this not only helps you to avoid silly mistakes but also signals to the examiner that you know what you are doing (which can be effective even if you haven't the foggiest idea what you are doing).
- If you get stuck, state in words what you are trying to do and move on (at A-level, you don't get credit for merely stating intentions, but university examiners are generally grateful for any sign of intelligent life).



Solving problems: some problems may have hidden depths.

10 Feedback

You will have plenty of opportunities to express your views. **Please use them, even if it is too late for you to benefit.** Lecturers want to give you the best course possible and welcome constructive feedback on both style and content. You must share the responsibility.

Towards the end of each course, the lecturer will distribute questionnaires and allow a little time in one lecture for you to fill them in. There will also be an electronic questionnaire at the end of the year. In addition, you can at any time e-mail the faculty hotline (`hotline@maths.cam.ac.uk`); hotline will either deal with your comment, or pass your e-mail to the relevant person.

In all cases:

- Try to be specific.
- Make comments which the lecturer can act on.
- Resist the temptation to be rude and/or clever.
- Resist the temptation to make personal comments about the lecturer and his or her appearance.[8]
- Bear in mind that it takes a great deal of time and effort and thought to produce a lecture course plus examples sheets and handouts.
- Remember that you are only giving opinion: even if you hated the course others may have enjoyed it.
- Be aware that a course you found dull or incoherent or difficult may seem very different when you come to revise it.
- Bear in mind that some aspects of the course may depend on the syllabus as well as on the way the lecturer chose to interpret it.

[8] Although if you have found some of his or her mannerisms distracting, you might want to mention it. A rare exception to the rule: when a professor in Texas took to lecturing in a cowboy hat, one of the evaluation forms had the comment 'All hat, no cattle'. Which is pretty funny and probably not unkind.

The following comments are taken from questionnaires from recent years ago (NB lecturers change after about 3 years). Judge for yourself which ones are useful. (and note that some tell us more about the student than about the course or lecturer).

Algebra and Geometry, Michaelmas 199N, Dr X:

1. Although the lecturer would plainly be happier teaching classics, he should remember that he is very obviously not qualified to do so and should perhaps attempt to provide clear teaching of mathematics instead. Valid proofs would also be welcomed.
2. Dr X was brilliant. [*NB: same lecturer and lecture course as the comment above.*]
3. Good introduction to group theory that possibly went a bit fast.
4. The course appears difficult to start with but is actually not too bad once read through a few times. The printed notes were a good idea.
5. Excellent lecturing and an interesting subject. I felt that some of the proofs were a little wordy and not precise enough.
6. I found the abstract ideas difficult to understand. The lecturer made it worse by using the word ‘trivial’ and giving no explanation when I thought it was far from trivial.

Mathematics for NST, Easter 199N, Dr S

1. The Fourier lectures were incomprehensible and going slower would have made no difference as the lecturer was crap.
2. Excellent course, excellent lecturer. Can he lecture all the courses?
3. The section on matrices was a bit slow.
4. The little anecdotes were very interesting/amusing.
5. Dr S was brilliant. Handouts very useful. Sometimes the writing on the overheads was unclear.
6. The lecturer left long pauses which tended to break my concentration.
7. Too many proofs; not enough worked examples.

11 And finally . . .

Mathematics is difficult. However, it is no more difficult than anything else – just difficult in a different way. Most people cannot read a chapter of a maths book and expect to end up with a decent understanding of the material. It has to be worked at, line by line. Don't be discouraged: it is just a different way of learning and no worse than the wading through of hundreds of often contradictory textbooks that many of your friends will have to suffer. And there are few more satisfying (academic) achievements than the successful proof of a tricky mathematical proposition.