23 Astrophysics

23.5 Ionization of the Interstellar Gas near a Star (8 units)

No knowledge of Astrophysics is assumed or required: all relevant equations are defined and explained in the project itself.

1 Introduction

The interstellar medium surrounding a hot star is ionized by the radiation from the star. In this project we calculate the size of the ionized region, and gain some insight into how its structure depends on the nature of the radiation from the star.

Assuming that there is a uniform, static, constant temperature gas surrounding a spherically symmetric star provides a good approximation, which keeps the essentials of the situation without allowing unnecessary distractions. Each gas element is assumed to be in ionization equilibrium, so the ionization rate for each atom is balanced by recombinations. We assume that the sole source of ionization is by absorption of radiation from the star giving a bound electron enough energy to escape from the atom. Recombination occurs when a free electron is captured by an ion with the creation of a photon. Since hydrogen is the most common element in the Universe, its behaviour will dominate in most cases, so we consider only a pure hydrogen interstellar medium.

2 Radiation from a star

The radiation from the star is specified as L_ν, the total energy output from the star per unit frequency ν per unit time, so the luminosity per unit frequency interval L_ν is expressed in W Hz^{-1}. The total energy output radiated from the star is the integral of this quantity over all frequencies, or $L = \int_0^\infty L_\nu \mathrm{d}\nu$. In some cases a star spectrum is reasonably well approximated by a black-body, so the radiation flux in a frequency interval $\mathrm{d}\nu$ at frequency ν emerging per unit area from the surface of the star where the temperature is T_* is given by

$$I_\nu \mathrm{d}\nu = \frac{2\pi h}{c^2}\frac{\nu^3}{\exp(\frac{h\nu}{kT_*}) - 1}\mathrm{d}\nu,$$

where h is Planck's constant, c is the velocity of light, and k is Boltzmann's constant (given below). Thus for a star of radius R

$$L_\nu = 4\pi R^2 \frac{2\pi h}{c^2}\frac{\nu^3}{\exp(\frac{h\nu}{kT_*}) - 1}.$$

Question 1 The sun has radius $R = 6.96 \times 10^8$ metres, and the total luminosity $L = 3.90 \times 10^{26}$ W. Show, using the above equation, that its surface temperature is close to 5800 K.

A 7 solar mass star has a surface temperature $T_* = 20,000$K and a luminosity $L = 4.0 \times 10^{29}$W. What is its radius? What is the radius of a 12 solar mass star which has a surface temperature $T_* = 25,000$K and a luminosity $L = 4.0 \times 10^{30}$W.

3 Equations for ionization and recombination

An element of gas at a distance r from the (centre of the) star will receive $\frac{L_\nu}{4\pi r^2}$ W m^{-2} Hz^{-1} if the radiation is not attenuated by any material between it and the star. The number of photons received per second per unit frequency interval is L_ν divided by the energy, $h\nu$ per photon. If these photons have frequency $\nu \geqslant \nu_0 = 3.29 \times 10^{15}$Hz they have enough energy to separate a hydrogen atom into a proton and an electron. The rate at which this happens depends on the frequency of the radiation, and is given by the photon rate per second $\times$ an absorption coefficient a_ν per hydrogen atom. For an energy flux $I_\nu(r)$, so photon flux $I_\nu(r)/h\nu$, the rate of ionization per unit volume is

$$n_{\mathrm{H}^0} \int_{\nu_0}^{\infty} \frac{I_\nu(r)}{h\nu} a_\nu \mathrm{d}\nu,$$

where n_{H^0} is the number density of neutral hydrogen atoms (i.e. number of neutral hydrogen atoms per cubic metre). The coefficient a_ν depends only on the atomic species being considered, and for neutral hydrogen

$$\begin{aligned} a_\nu &= a_{\nu_0} \left(\frac{\nu_0}{\nu}\right)^3 \quad \text{for } \nu \geqslant \nu_0 \\ a_\nu &= 0 \quad \text{for } \nu < \nu_0, \end{aligned}$$

where $a_{\nu_0} = 6.3 \times 10^{-22}$ m^2. If absorption can occur at the gas element, it can also occur in all the elements between the star and the one under consideration. As we go along a path dr the radiation is attenuated, so at a given frequency ν,

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}r} = -n_{H^0} a_\nu I_\nu,$$

and so

$$I_\nu(r) = I_\nu(R) e^{-\tau_\nu},$$

where τ_ν is defined by

$$\frac{\mathrm{d}\tau_\nu}{\mathrm{d}r} = n_{H^0}(r) a_\nu$$

and $\tau_\nu(R) = 0$. τ_ν is referred to as the *optical depth* at the frequency ν. There is no redistribution of the photons in frequency (they are effectively absorbed from the point of view of this calculation), so we can determine τ_ν from

$$\frac{\mathrm{d}\tau_{\nu_0}}{\mathrm{d}r} = n_{H^0}(r) a_{\nu_0}$$

and

$$\tau_\nu = \tau_{\nu_0} \left(\frac{\nu_0}{\nu}\right)^3.$$

This absorption of radiation and the ionization it causes must be balanced by recombination of protons and electrons at the same rate per unit volume. This rate depends on the number densities of the protons (n_p) and the electrons (n_e), their relative velocity and a velocity-dependent cross-section for the interaction which has to be integrated over the velocity distribution at whatever temperature the gas is at. These velocity-dependent terms are combined into recombination coefficients α_B which are tabulated for various gas temperatures T:

T (K)	α_B
5 000	4.54×10^{-19} m^3 s^{-1}
10 000	2.59×10^{-19}
20 000	2.52×10^{-19}

Then we are in a position to write down the ionization balance equation

$$n_{\mathrm{H}^0} \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu e^{-\tau_\nu} \mathrm{d}\nu = n_p n_e \alpha_B(T), \tag{1}$$

and subsidiary equations

$$\frac{\mathrm{d}\tau_{\nu_0}}{\mathrm{d}r} = n_{H^0}(r) a_{\nu_0}, \tag{2}$$

$$\tau_\nu = \tau_{\nu_0} \left(\frac{\nu_0}{\nu}\right)^3, \tag{3}$$

$$n_p = n_e, \tag{4}$$

$$n_p + n_{H^0} = n_H, \tag{5}$$

which govern the ionization balance for the hydrogen-filled interstellar medium near a star.

[We have omitted a step here, and assumed that the energy released by recombination does not give rise to an ionizing photon. Often it does, but we assume that all this does is cause another ionization nearby until recombination occurs to an upper level in the hydrogen atom, and then the energy is lost from the system through radiation at frequencies less than ν_0. The net result is a change in the effective recombination coefficient, to the one quoted here. Those interested in more details will find them in Osterbrock's book (1989)]

4 Ionization near stars

A typical interstellar medium hydrogen number density is $n_H = 10^6$ m^{-3}, and a typical temperature is $T = 10^4$K.

Question 2 Write a program to solve the ionization equations to obtain the neutral hydrogen and proton densities as a function of distance from the centre of the star, assuming that the interstellar gas has a constant temperature and density. Note that the coefficients involved have large powers of 10, so for some compilers a rescaling of variables may be desirable. Describe any transformations used. Are there any advantages to using τ_{ν_0} instead of r as the radial coordinate?

Now apply this program to some realistic cases:

Question 3 Determine the ionization and neutral fractions of hydrogen as a function of distance from the star with surface temperature $T_* = 20,000$K and luminosity 4×10^{29}W for an interstellar gas density $n_H = 10^6$ m^{-3} and $T = 10^4$K, and plot the results. Show in particular that at some radial distance, r_1, there is quite a sharp transition from the gas being mostly ionized to mostly neutral. What is the value of r_1, the distance from the centre of the star where $n_p = n_{H^0}$? Give your answers to two significant figures.

Repeat the calculation for gas of the same temperature and density but with the 12 solar mass star at the centre and again with the Sun as the central star. Also, compute the ionization fractions for all three stars in the cases that the gas temperature is 5000K and 20, 000K. Comment on any similarities or differences between the three cases.

Provide plots of the nine cases (three stellar masses and three gas temperatures) and a table containing the values of r_1 for each case.

4.1 An approximation to r_1

Equation (1) can be integrated over volume from $r = 0$ to $r = \infty$ by using the definition of τ_ν to replace dr and assuming that the recombination term is well approximated by $n_p = n_e = n_H$ for $r \leqslant r_1$ and $n_p = n_e = 0$ for $r > r_1$. This r_1 is called the Strömgren radius.

Question 4 Show that, under these circumstances,

$$Q(H) \equiv \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} \mathrm{d}\nu = \frac{4\pi}{3} r_1^3 n_H^2 \alpha_B,$$

where $Q(H)$ is the total number of ionizing photons emitted by the star per second.

Calculate the values of $Q(H)$ for the cases given above and compare the r_1 determined from this approximation with the values you have computed for $T = 10,000$K.

5 The effect of a quasar on the host galaxy

The energy output from a quasar is very different from that of a star, both in intensity and frequency dependence. A bright quasar has energy output of 4×10^{39}W, and resides in the centre of a galaxy of radius 3×10^{20}m (= 30,000 light years).

Question 5 If the frequency dependence of the quasar luminosity at frequencies $\nu \geqslant \nu_0$ is given by

$$L_\nu = 10^{24} \left(\frac{\nu}{\nu_0}\right)^{-1.4} \exp\left(-\frac{\nu}{10\nu_0}\right),$$

determine whether or not any of the interstellar gas in the galaxy has neutral fraction $n_{H^0}/n_H > 0.5$. (Assume that the gas temperature is 10^4K, and density 10^6 hydrogen atoms per cubic metre. Assume also that there is no gas within 10^{18}m of the quasar position, so start the computation there.)

Tabulate the results for the hydrogen neutral fraction as a function of distance from the quasar.

Useful constants:

velocity of light	$c = 2.998 \times 10^8 \text{ms}^{-1}$
Planck's constant	$h = 6.626 \times 10^{-34}$Js
Boltzmann's constant	$\mathrm{k} = 1.381 \times 10^{-23}\text{JK}^{-1}$

References

[1] Osterbrock, D.E., 1989. *Astrophysics of Gaseous Nebulae and Active Galactic Nuclei* University Science Books: Mill Valley, CA (Especially chapter 2).

