Topics in Set Theory (L24)

B Löwe

This course covers advanced topics in set theory, focusing on meta-mathematical techniques such as inner models and forcing.

Set theory and logic are intrinsically intertwined since the most interesting results in set theory are independence results showing that natural questions in set theory are not solvable using the standard axiomatic system of Zermelo-Fraenkel set theory with choice ZFC.

The most famous of these natural questions is Cantor's continuum hypothesis CH, "every uncountable set of reals is equinumerous to the set of all real numbers" or, equivalently, $2^{\aleph_0} = \aleph_1$. This question was elevated to the status of the foremost mathematical problem for the 20th century by David Hilbert in his address to the *International Congress of Mathematicians* in Paris in the year 1900. In 1938, Kurt Gödel proved that CH cannot be disproved in ZFC (inventing and using the method of inner models); in 1963, Paul Cohen proved that CH cannot be proved in ZFC (inventing and using the method of forcing). Together, these results show that CH is independent from ZFC.

We shall treat several of the following topics:

Model theory of set theory. Models of set theory. Absoluteness. Simple independence results. Ranks. Reflection principles.

Large cardinals. Introduction to large cardinals. Inaccessible cardinals. Measurable cardinals. Ultrapowers. Scott's theorem.

Inner models. Definability. Ordinal definability. Constructibility. Condensation. Gödel's proof of the consistency of CH.

Forcing. Generic extensions. The forcing theorems. Adding reals; collapsing cardinals. Cohen's proof of the consistency of $\neg\mathsf{CH}$.

Pre-requisites

The Part II course *Logic and Set Theory* or an equivalent course is essential.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.