Topics in Random Graphs (L16)

Graduate Course

Richard Montgomery

Random Graphs have been a fundamental object of study in combinatorics since the pioneering work of Erdős and Rényi in the 1960’s. Not only do random graphs have many interesting properties, but their study provides a rich space in which to mix combinatorial and probabilistic ideas. In this course, we will explore some of these ideas.

For any given (monotone) graph property (such as connectivity), if we steadily increase the density of edges in a random graph then at a certain point - known as the threshold for the property - the random graph will suddenly become very likely to have this property. We will determine the thresholds of a range of different graph properties, and also prove various hitting time results, where a complex property is shown to almost surely hold in a random graph if (and only if) some simpler property holds. The material covered will include coupling probability models, sprinkling edges, conditioning arguments, Pósa’s rotation technique, vertex absorption, and other methods.

Pre-requisites

We will assume only some very basic notions of probability and graph theory.

Literature