This course will be an introduction to Itô calculus.

- **Brownian motion.** Existence and sample path properties.

- **Stochastic calculus for continuous processes.** Martingales, local martingales, semi-martingales, quadratic variation and cross-variation, Itô’s isometry, definition of the stochastic integral, Kunita-Watanabe theorem, and Itô’s formula.

- **Applications to Brownian motion and martingales.** Lévy characterization of Brownian motion, Dubins-Schwartz theorem, martingale representation, Girsanov theorem, conformal invariance of planar Brownian motion, and Dirichlet problems.

- **Stochastic differential equations.** Strong and weak solutions, notions of existence and uniqueness, Yamada-Watanabe theorem, strong Markov property, and relation to second order partial differential equations.

- **Stroock–Varadhan theory.** Diffusions, martingale problems, equivalence with SDEs, approximations of diffusions by Markov chains.

**Pre-requisites**

Knowledge of measure theoretic probability as taught in Part III Advanced Probability will be assumed, in particular familiarity with discrete-time martingales and Brownian motion.

**Literature**

3. I. Karatzas and S. Shreve *Brownian Motion and Stochastic Calculus.* Springer. 1998
5. D. Revuz and M. Yor, *Continuous martingales and Brownian motion.* Springer. 1999