Shimura varieties, some examples (L24)

*Non-Examinable (Part III Level)*

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Modular curves — spaces like the quotient of the upper half plane by $SL_2(\mathbb{Z})$ — are ubiquitous in number theory and geometry. They parameterise elliptic curves with level structure, are defined over number fields, and are a source of Galois representations.

Shimura varieties are generalisations of modular curves. They are an immensely rich class of examples which are of significance in geometry, where they parameterise certain Hodge structures, and in number theory, where they are a source of certain Galois representations.

This course will be cover some examples and basic theorems about Shimura varieties. Some of the results will be about arbitrary locally symmetric varieties.

I am really not an expert on this, and I’m not yet sure what theorems we will cover. Some topics we may include are: Tamagawa number theorems, complex multiplication, Gross-Kohnen-Zagier formulas, explicit automorphic forms, as well as examples of the relation to the Langlands program and to moduli problems in algebraic geometry.

Most of the theorems are already substantial and deep in the modular curve case.

**Pre-requisites**

This course assumes you know nothing but can do anything. The Part II Number Fields course, algebraic geometry, and the basics of Lie algebras will all be helpful, but aren’t required.

If you already have an opinion on Shimura varieties, global or local, you probably know more about these matters than I do. Please suggest essential theorems we need to cover, as well as telling me how I am doing it wrong.... (and do come along to class occasionally, and contribute).

**Literature**

There is an immense literature on Shimura varieties. We will slowly pick our way through parts of it.