Functional Analysis (M24)
András Zsák

This course covers many of the major theorems of abstract Functional Analysis. It is intended to provide a foundation for several areas of pure and applied mathematics. We will cover the following topics:

Hahn–Banach Theorem on the extension of linear functionals. Locally convex spaces.

Duals of the spaces $L_p(\mu)$ and $C(K)$. The Radon–Nikodym Theorem and the Riesz Representation Theorem.

Weak and weak-* topologies. Theorems of Mazur, Goldstine, Banach–Alaoglu. Reflexivity and local reflexivity.


Some additional topics time permitting. For example, the Fréchet–Kolmogorov Theorem, weakly compact subsets of $L_1(\mu)$, the Eberlein–Šmullian and the Krein–Šmullian theorems, the Gelfand–Naimark–Segal construction.

Pre-requisites

Thorough grounding in basic topology and analysis. Some knowledge of basic functional analysis and basic measure theory (much of which will be recalled either in lectures or via handouts). In Spectral Theory we will make use of basic complex analysis. For example, Cauchy’s Theorem, Cauchy’s Integral Formula and the Maximum Modulus Principle.

Literature


Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term. There will be some material as well as examples sheets and announcements available at [www.dpmms.cam.ac.uk/~az10000/](http://www.dpmms.cam.ac.uk/~az10000/)