

# **Oxbridge PDE Conference 2012**

*MR2, Centre for Mathematical Sciences, University of  
Cambridge*

*17-18 April 2012*

**Convenors:**

Professor Mihalis Dafermos, DPMMS

Dr Clément Mouhot, DPMMS

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**Conference website:**

<http://www.maths.cam.ac.uk/postgrad/cca/oxbridgepde2012.html>

**All lectures will take place in MR2**

Centre for Mathematical Sciences

University of Cambridge

Wilberforce Road

Cambridge

CB3 0WA

## Tuesday 17th April

**10am**            **Professor Adrian Constantin, King's College London**  
*Particle trajectories beneath irrotational travelling water waves*

We describe the pattern of the particle trajectories beneath a travelling wave moving at the surface of water in irrotational flow and with a flat bed, with no underlying current, both in the setting of periodic waves (Stokes waves) and in the setting of solitary waves.

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**11am**            **Coffee**

**11.30am**        **Dr Neshan Wickramasekera, DPMMS**  
*Higher multiplicity in minimal varieties*

The focus of the talk will be on the nature of singularities of minimal varieties (i.e. stationary points of the area functional) of a Riemannian manifold. A quick survey of some old results concerning singularities, most of which are valid for "multiplicity 1 classes" of minimal varieties, will be followed by a discussion of some recent progress concerning the size and the structure of the set of branch points, i.e. those singularities that arise in the presence of higher multiplicity.

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**12.30pm**        **Lunch in Central core**

**2pm**             **Professor Gui-Qiang Chen, University of Oxford**  
*Free Boundary Problems in Nonlinear Hyperbolic Conservation Laws*

In this talk we will present several free boundary problems for the stability of multidimensional discontinuities and the existence of fundamental wave patterns in solutions to nonlinear hyperbolic conservation laws, especially in fluid mechanics and MHD. The discontinuities include shock waves, vortex sheets, and entropy waves. Some recent developments will be reviewed and discussed. Further trends, perspectives, and open problems in this direction will be also addressed.

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**3pm**             **Professor Gregory Seregin, University of Oxford**  
*Ancient solutions to the Navier-Stokes equations in half-space*

A conception of ancient (backward) solutions is discussed in connection with a question about finite time blowup in the initial boundary value problem for the Navier-Stokes equations in half-space.

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4pm

**Coffee**

4.30pm

**Dr Laura Caravenna, University of Oxford**

*Joint works on Continuous solutions to a balance equation*

The talk is concerned with continuous solutions to a scalar, 1D balance law having bounded source term. We will discuss the correspondence between Eulerian and Lagrangian formulation, mostly focusing on the simple equation  $u_t + [u^2/2]_x = g$  with  $g$  bounded and assuming  $u$  continuous but neither Sobolev nor BV. This is strictly related to a characterization of intrinsic Lipschitz graphs in the sub-Riemannian Heisenberg groups, after the characterization which had already been given of intrinsic regular graphs. The talk will be mainly based on collaborations with G. Alberti, S. Bianchini, F. Bigolin, F. Serra Cassano.

[Laura.Caravenna@maths.ox.ac.uk](mailto:Laura.Caravenna@maths.ox.ac.uk)

## Wednesday 18th April

10am

**Dr Andre Neves, Imperial College London**

*Min-max methods and the Willmore conjecture*

In 1965, Willmore conjectured that for every torus in space, the integral of the mean curvature squared is bigger or equal to  $2\pi^2$ . I will talk about how to prove this conjecture using min-max methods. This is joint work with Fernando Marques.

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11am

**Coffee**

11.30am

**Professor John Ball, University of Oxford**

*Smooth topology-preserving approximations of rough domains*

The talk concerns bounded domains with continuous boundary. We study how the corresponding "good direction", with respect to which the boundary is locally a graph of a continuous function, varies in a neighbourhood of the boundary, and thus show how such domains can be approximated both from the inside and the outside by topologically equivalent smooth domains. The good directions form a globally-defined field that carries some topological information about the domain, which we explore. Finally we describe a surprising consequence of the study, that the domain has portions of the boundary with better regularity. This is joint work with Arghir Zarnescu.

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12.30pm

**Lunch in Central core**

2pm

**Professor Mihalis Dafermos, DPMMS**

*The black hole stability problem*

A fundamental open problem in general relativity is the nonlinear stability of the Kerr family of "black hole" spacetimes as solutions to the Einstein vacuum equations. The problem has received considerable attention in recent years, but even its linear aspects are still only partially understood. In this talk, I will introduce the problem from the perspective of a PDE audience and review its current status.

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3pm

**Dr Jacques Smulevici, Université Paris-Sud 11**

*Non-linear stability of Schwarzschild-AdS for the spherically symmetric Einstein-Klein-Gordon system*

Joint work with Gustav Holzegel (Princeton University). One of the most

interesting objects of general relativity are the so-called black hole spacetimes. Mathematically, they can be represented as Lorentzian manifolds possessing certain global geometrical properties and whose Lorentzian metrics satisfy a set of hyperbolic PDES (the Einstein equations). Since the Einstein equations are evolution equations, a natural question is to determine which solutions are stable or not from the point of view of the initial value problem. After an introduction to the study of the Einstein equations, I will present one particular black hole solution, namely the Schwarzschild-AdS solution, and sketch a proof of its stability for the spherically symmetric Einstein-Klein-Gordon system. If time permits, I will also discuss a linear decay result (without symmetry) on Schwarzschild-AdS and Kerr-AdS.

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**4pm**

**Coffee and close**