Quantum Information Theory

Cristina Cirstoiu - a third year undergraduate reading the mathematical tripos. During the summer before my third year, I worked with Dr. Eugene Lim on a research project in DAMTP. The first part of my project involved reading about quantum information theory from an introductory level up to the point where I was able to understand some fairly recent research articles on this topic.For the remaining time, I focused on black holes from a quantum information perspective.

Quantum information is a relatively new and rapidly growing field which exploits how the laws of quantum mechanics can be applied to improve how information is transmitted,aquired and processed.It is a fascinating area due to its interdisciplinary nature, lying at the interplay of physics,mathematics, computer science and engineering.

One of the many research areas in which quantum information can bring useful insights is the foundations of physics. In particular a first example of this is a recent article of L.del Rio et al [1] that I've been studying.It analyses how entanglement can be used as a physical resource,by providing an operational meaning to the concept of negative conditional entropy.

They did this by extending Landauer's principle (which says that the work required to erase a system by an observer is proportional to the entropy of the system, conditioned on the knowledge of the observer e.g. if one knows that a system is already in a pure state then nothing has to be done!) to the full quantum case, where the observer's memory is made up of qubits that store information via entanglement.Therefore since the conditional entropy can have a negative value,one can gain work by erasing a system.Through the process, all correlations are broken,and this results in negative work,whilst the actual erasure requires work (as in the classical case).The overall effect is that work can be gained.This is very useful as it provides a connection between thermodynamics and information theory.

Secondly, I have been looking at black holes and how information encoded in the Hawking radiation can be recovered.

Due to quantum fluctuations at the event horizon of a black hole, pairs of maximally entangled particles are created.One of the particles escapes the gravitational pull,forming the so-called Hawking radiation.

It is interesting to note, as it's discussed in [2], that if Alice throws some qubits in the black hole, then Bob could start retrieving that information from the emitted radiation quickly after he receives more than half of the black hole's initial qubits.This is shown by finding appropriate bounds on the difference in the fidelity of the information before and after retrieval, when compared to a reference system.

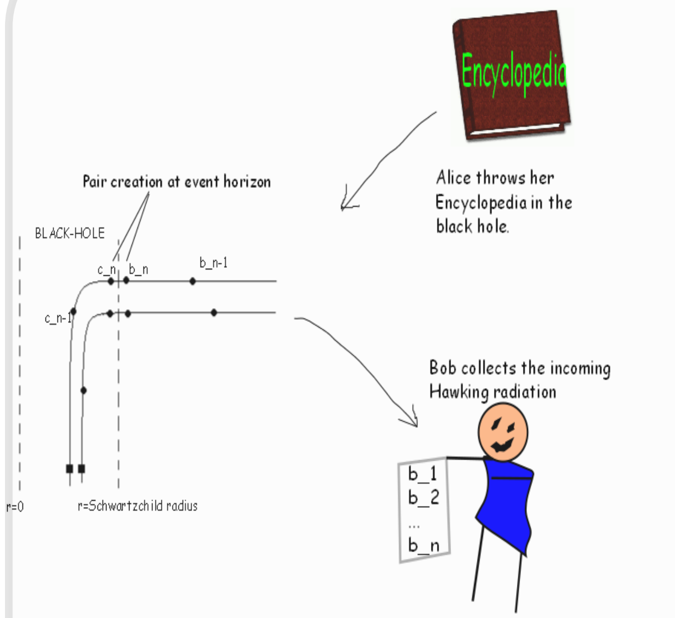
As it can be seen in the picture, the space-like slice streches and as new pairs are created at the event horizon, the outgoing radiation (b_n particles) gets further away, whilst the c_n form the infalling matter.

As the black hole continues to evaporate, its final state could be either:

-mixed state: the black hole evaporates completely. The quanta b_n (having Von Neumann entropy n log 2) end up in a fully mixed state since there is nothing left in the black hole to be entangled with. This implies that a pure state has evolved into a fully mixed state, hence contradicting unitarity of quantum mechanics.

-remnant: the black hole evaporates until its mass reaches a certain value and then it stops.The remnants would be entangled with the quanta b_n outside.However, this is not predicted by current theories.

Therefore, both of the possible final states lead to situations which cannot be solved through classical theories, although several resolutions (i.e string theory,holographic principle) have been proposed. The final answer will probably be provided by quantum gravity and one can hope that in the future, quantum computers will be able to simulate this physical theory at a sub-Planckian scale.



$$|\psi_{Black\text{-}hole}\rangle = \frac{|0_{b\_n}0_{c\_n}\rangle + |1_{b\_n}1_{c\_n}\rangle}{\sqrt{2}} \otimes \ldots \otimes \frac{|0_{b\_1}0_{c\_1}\rangle + |1_{b\_1}1_{c\_1}\rangle}{\sqrt{2}} \otimes |\psi_{initial}\rangle$$

From a personal perspective, working on this project has been an invaluable experience; challenging yet rewarding.

I had the opportunity to get a flavour of what academic research means,to learn and develop a deep understanding of a subject I enjoy,to gain new skills from which I'll benefit both in my studies and in any further research.

Also, I believe it is an exciting field to work in, as theoretical work is often backed up by cutting edge experiments.

[1]L del Rio et al. arXiv:1009.1630v2
[2]J. Preskill,P Hayden arXiv:0708.4025v2
[3]S.Mathur arXiv:0909.1038v2