University of Cambridge
Faculty of Mathematics
Summer Research Projects

Modified Gravity in Cosmology and Astrophysics

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Why Modified Gravity?

The standard ΛCDM (Λ-Cold Dark Matter) cosmology uses a cosmological constant to give the current acceleration of the Universe.

The energy stored in this constant is called dark energy, but the scale of magnitude of the energy density of this dark energy has no good explanation. Other models have therefore been proposed instead in order to explain the acceleration of the Universe.

Scalar field models

The Einstein equations of the ΛCDM cosmology can be derived from an action:

$$\int d^4x\sqrt{-g}\frac{M_{Pl}^2}{2}(R-\Lambda)$$

where g is the determinant if the metric, M_{Pl} is the Planck mass and R is the Riemann trace. When a scalar field is included, the action becomes:

$$S=\int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R+\frac{M_{Pl}^2}{2}\phi^2+V(\phi)\right)+S_m(A^2(\phi)g_{\mu\nu},\phi)$$

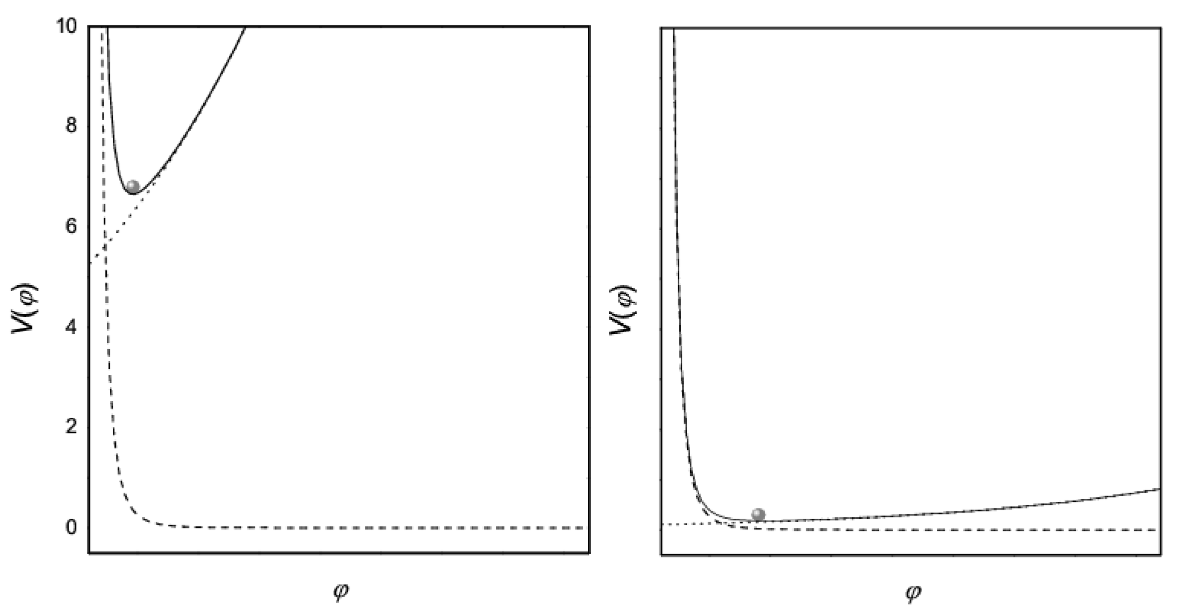
Here ϕ is the scalar field, $V(\phi)$ is the potential and $A^2(\phi)$ gives the coupling to matter, as S_m is the matter action.
This type of model is determined by the specific choice of $V(\phi)$ and $A(\phi)$. The scalar field gives an effective potential

$$V_{eff}=V+(A-1)\rho$$

where ρ is the matter density. The field seeks to minimise this effective potential.

Scalar fields cause a fifth force to act between objects when the field is couples to ordinary matter, which would have observable consequences.

Screening mechanisms



The Chameleon model: the field in dense environments is on the left, with the minimum of the field close to zero, and in less dense environments on the right.

For the model to be viable, it must not violate any of the results of observations we make- the fifth force caused by the scalar field must be too small to be noticed where we can make measurements.

Screening Mechanisms cont'd

Several models depend on the environment to achieve this. For the Chameleon model [1], the field is strongly coupled to matter, and the mass of the field is much larger in the laboratory so it will have little effect in comparison to it being almost massless on cosmological scales.

Another type of model is the environmentally dependent dilaton [2], where the coupling to matter $\beta=\frac{d\ln A}{dt}$ depends on the field, and turns off in high density environments.This is an example of the Damour-Polyakov mechanism which comes from string theory.

Particular model

The particular model is not environmentally dependent, so the screening mechanism works in a different way to above. It uses:

$$A(\phi)=1+\frac{\alpha}{2}\phi^2$$

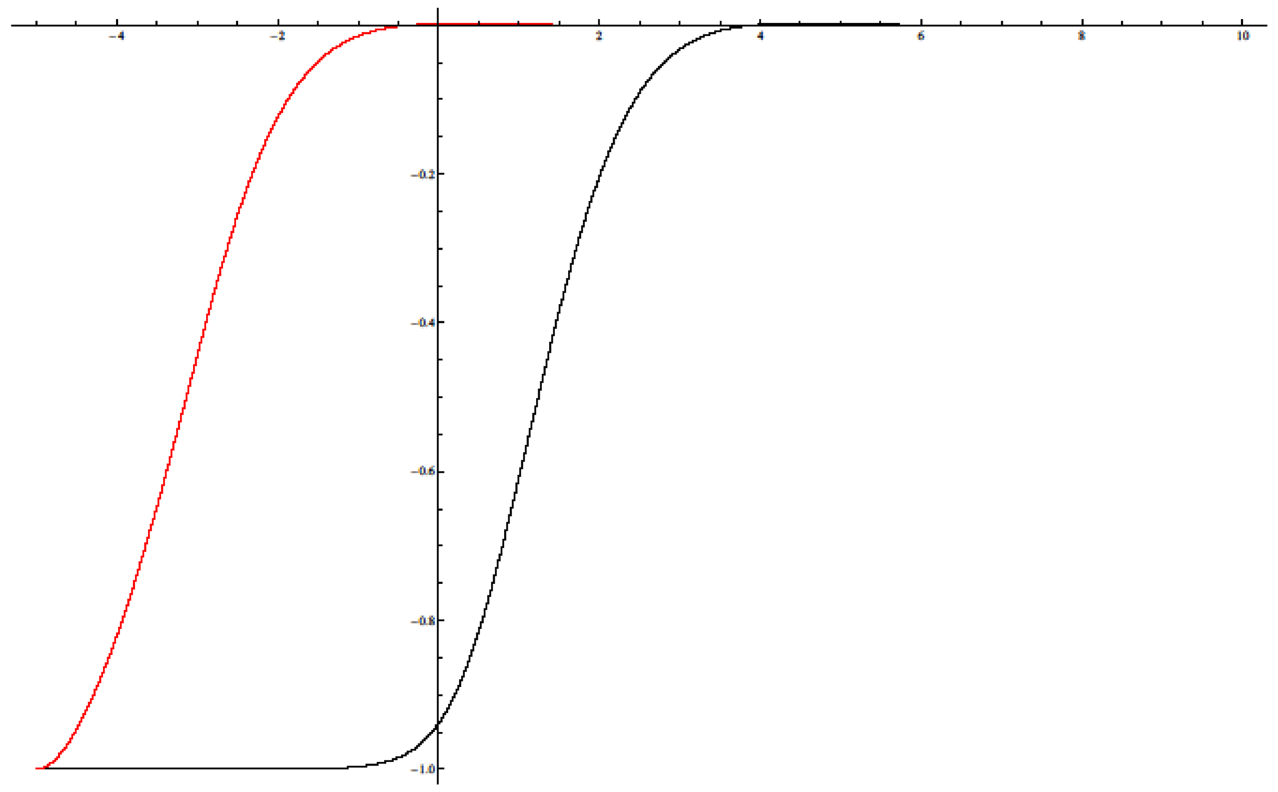
$$V(\phi)=m^2M_{Pl}^2(1+\frac{\lambda}{2}\phi^2)$$

.

There are several ways to check the model is a possibility.

Testing- Cosmology

The model must be experimentally viable. The first way check is to examine the cosmology of the model. The FRW metric for a spatially homogeneous isotropic universe uses a scale factor for the universe $a(t)$. The modified Einstein equations derived from the action can be solved to find the Hubble constant $H=\frac{\dot{a}}{a}$.



The above figure shows $H(t)$ as a function of time for different values of α. The value of H today is known, so the equations for the model can be solved in time, starting in the past, to find the value of $H(t)$ today and compare it to this value.

The model depends on several parameters: α,λ and the mass of the scalar field, m. Solving the equations for the model numerically and then comparing to the known values gives the possible values of these parameters so that the values match.

Solar System Tests

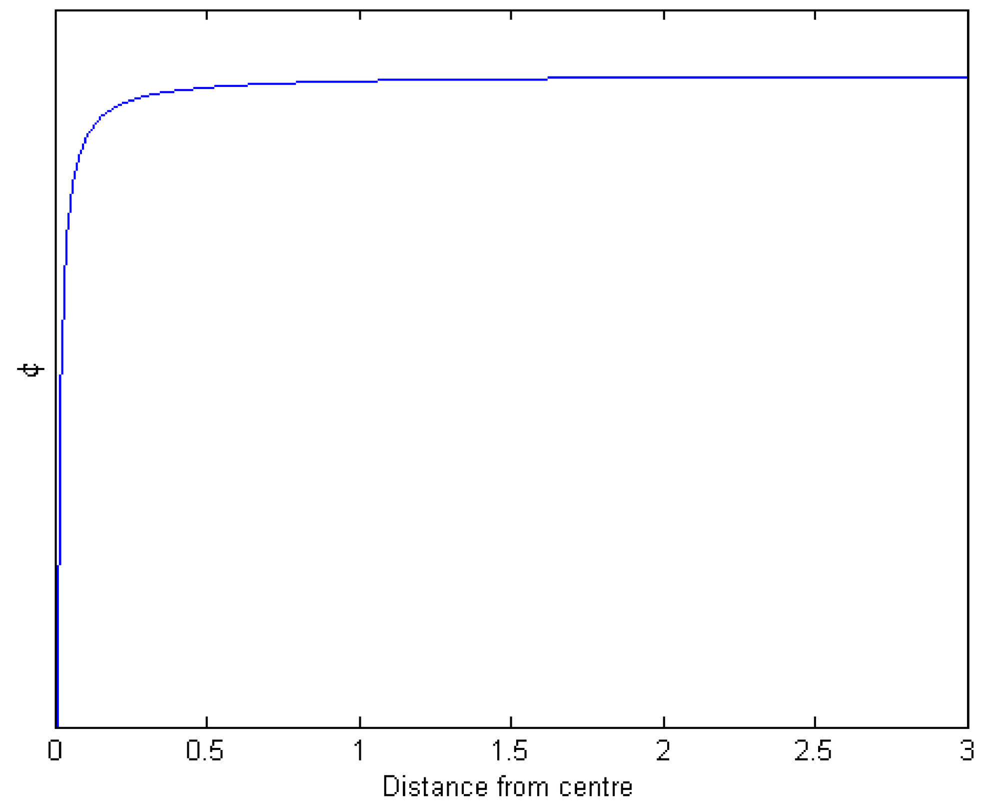
There are several solar system tests which give bounds on the parameters:
The Cassini experiment, which measures the frequency shift of photons (from the curvature of space-time) near the sun, gives one bound of

$$\beta<10^{-5}$$

Lunar Laser Ranging, which imposes limits on any violation of the equivalence principle, also provides more constraints on the parameters.

Spherical objects

The spherical profile of the scalar field at a fixed time time shows how the field changes with distance from a spherical object (such as the Earth or Sun). It useful to see how the field drops off away from the object. The graph below shows the field profile around such a spherically symmetric object, assuming the background is perturbed Minkowski space.



The figure above shows how the field drops off around a spherically symmetric object, tending towards the background value of the field.

Conclusions

It is possible to find values of the parameters that satisfy these tests, and there are then more features of the model to examine, such as the behaviour near black holes.

References

[1] J. Khoury and A. Weltman, Phys. Rev. D69, 044206 (2004)
[2] P. Brax, C. van de Bruck, A. C. Davis and D. J. Shaw Phys. Rev. D82, 063519 (2010)