Pooling Systems with Specialisation and Discretionary Task Completion

Mitchell Gooding

Judge Business School, University of Cambridge

The Problem

This project applied aspects from queueing theory to the specific setting of an accident and emergency (A&E) department in a hospital.

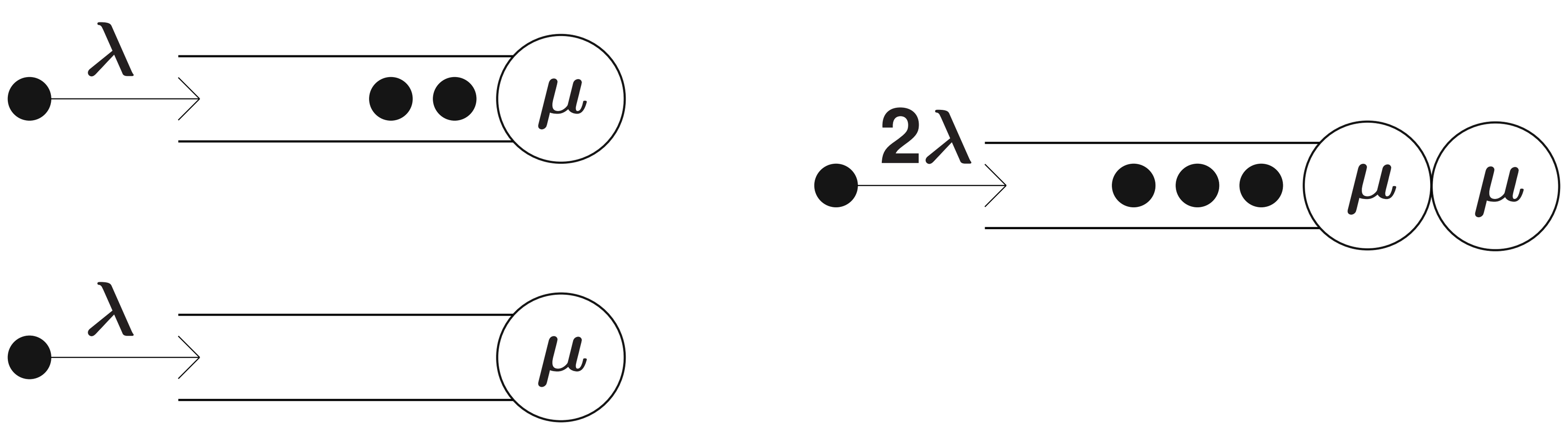
- This setting has a particular need for mathematical analysis, due to the National Health Service's goal of limiting 95% of A&E patient waiting times to four hours or less.

It was assumed that patient arrivals and service times were independent Poisson processes (the queues were modelled as $M/M/k$). We focused on three variations of the $M/M/k$ queue which empirical studies suggest are important to the A&E setting. These were pooling, specialisation and discretionary task completion (DTC).

Pooling

A hospital can choose to pool its resources into one general department, or create wards dedicated to a single type of patient, such as the paediatric unit at Addenbrooke's Hospital in Cambridge.

$2 \times M/M/1$ vs $1 \times M/M/2$

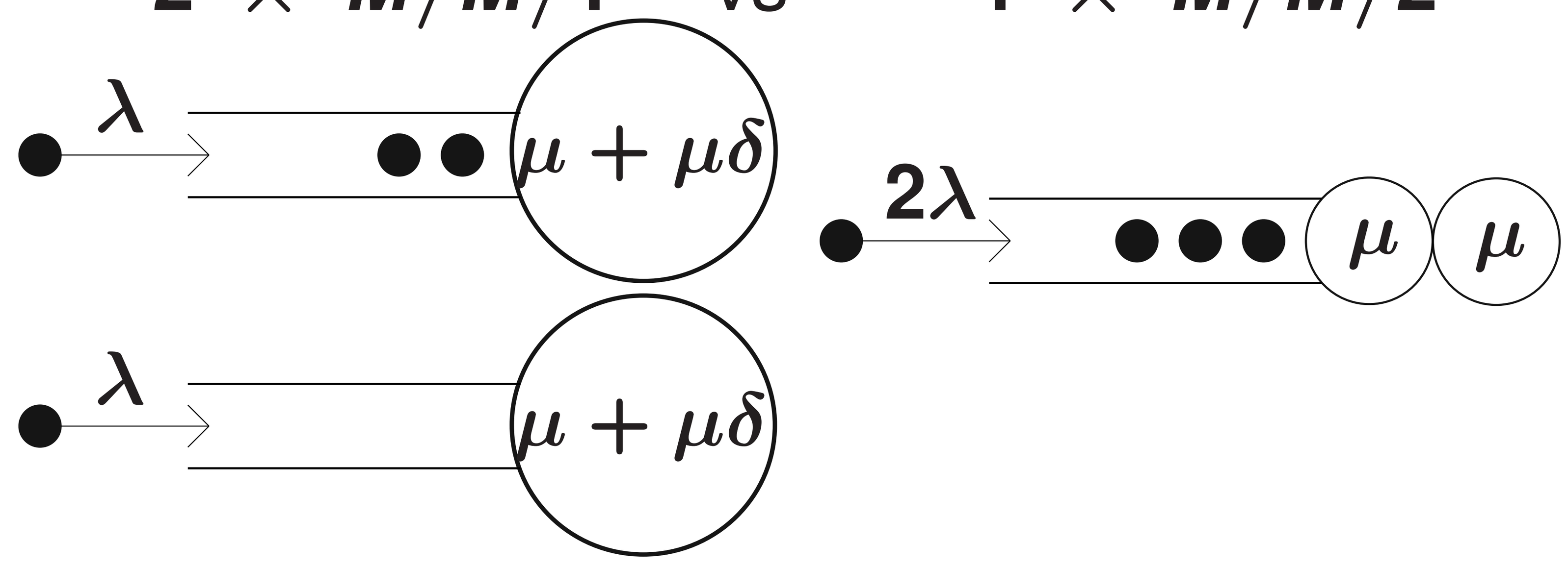


- Pooling performs better if service rates are identical.
- Not necessarily true if service rates are different [Benjaafar, 1995].

Specialisation

Dedicated queues may experience improved performance, perhaps due to specialised doctors and facilities. This is modelled by increasing the service rate by a factor of δ.

$2 \times M/M/1$ vs $1 \times M/M/2$



- When δ is 0, pooling is better.
- When δ is very large, dedication must be better.

Discretionary Task Completion

Discretion in task completion occurs when the value of the task gradually increases over time.

- Manufacturing tasks are often not DTC, as they only have value once the product is finished.

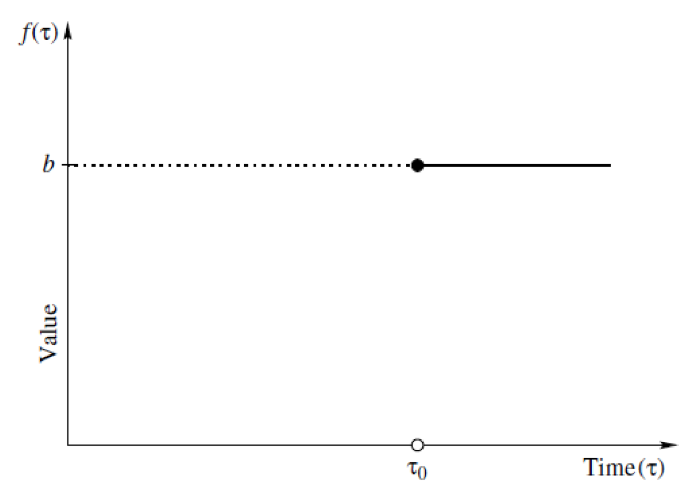


Figure: No Discretion in completion [Hopp, et al., 2007]

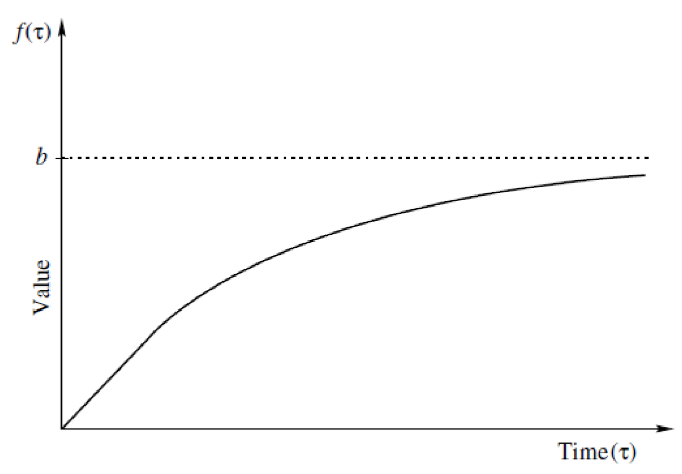


Figure: Discretion in completion [Hopp, et al., 2007]

We modelled DTC in the A&E setting by replacing a constant medical service rate with service rate which increases by a factor of Δ once the queue length exceeds a threshold.

Results

Mathematical models were found in literature (possibly requiring slight modification) for all but one of the queues outlined above. Unfortunately this was the model where all three effects were present, arguably the most important case. Comparisons made between the modelled cases usually confirmed the intuitive result. A nontrivial $\delta(k, \lambda/\mu)$ was found separating pooling-optimal and dedication-optimal areas.

References

- Wallace J. Hopp, Seyed M. R. Iravani, Gigi Y. Yuen (2007), *Operations Systems with Discretionary Task Completion*, *Manage. Sci.* 53(1), 61 – 77.
- Saifallah Benjaafar (1995), *Performance Bounds for the Effectiveness of Pooling in Multi-Processing Systems*, *Eur. J. Oper. Res.* 87(2), 375 – 388.