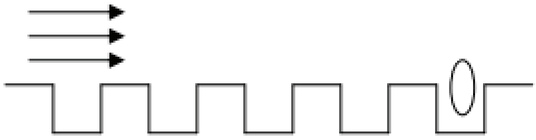
Fluid Flow over Aircraft Wings

James Kilbane, a third year undergraduate reading the mathematical tripos. I worked on a research project with Dr Stephen Cowley, the main idea of which was to research how air flowed over an aircraft wing which had been modified. The idea was that the aircraft wing had divots in, and to see how this affected the flow over the wing, primarily, to see if this configuration was stable. The picture below shows the configuration investigated.



The difficulty in solving this state comes from how the flow behaves in between the divots. In the entirely flat case, the answer is simple, and was solved over 200 years ago. However, with the divots the flow is disturbed, and patterns are created like the circles shown. This complicated behaviour becomes difficult to model and predict, and we need to turn to a numerical solution

The numerical solution, first, takes several simplifying approximations. The first is an approximation known as 'triple deck theory'. This theory considers flow around an object in three parts, the flow closest to the object, the flow away from the object and (roughly) the flow in the middle. The flow away from the object is idealized, and said to be the flow if no object were there, the flow closest to the object is governed almost entirely by the presence of the object, and the middle merges these two behaviours.

This approximation aids in solving the equations, it allows us to focus almost entirely on the behaviour near the object; this is the most interesting thing to see. Here, it simplifies the equations, allowing the programming to be ultimately simpler.

Another approximation we make is to assume that the wing extends to infinity. This approximation assumes we are simply looking in the middle of the wing, and the effects from the edges are somewhat negligible (eg the divot size is small in comparison to that of the wing.) This allows us to bring to force a theory known as Spectral Theory, one that makes the programming simpler, and confers more accurate results.

The last approximation is to treat the entire system as a two dimensional plane. This, again, makes everything easier, and can be justified thus, we are looking in the middle of a large wing.

After making these two approximations, we then have to solve the resultant equations, the Navier Stokes equations, a set of non linear differential equations that we have to solve. The fact that we are applying Spectral Methods allowed me to first of all convert everything into Fourier Series. After expanding and collecting terms, I was left with coupled differential equations, retaining their non linearity from Navier Stokes. These were then cast to difference equations using finite differences, and the boundary conditions were supplied by the approximations, Triple Deck Theory and a piece of asymptotics to justify the final conditions. This then gave a set of non linear equations to solve.

This is then solved through (primarily) Newton Raphson iteration. This is a technique which allows you to solve large sets of non linear equations, if one can invert a large matrix (the Jacobian of the system). This leads to more technical issues, which can be resolved by appealing to the sparsity of the system. This then produces the graph, as shown below, demonstrating the flow over the wing.

This flow shows some of the complexities inherent in the flow, eg the way the flow is almost sucked into the divots on the left hand side. It also shows how the flow is shifted all throughout by the presence of the divots (this follows from the assumption of incompressibility.) This, coupled with checks on the results, formed the rest of my time.